Calculation of the reliability function and the remaining life for equipment with unobservable states

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Abstract: This article proposes a model to calculate the reliability function and the expected remaining life of a piece of equipment when its state (degradation condition) is not perfectly observable. The Proportional Hazards Model (PHM) of Cox is used to model the failure rate of the equipment. According to the PHM, the failure rate of the equipment is a multiplicative function of a baseline function, which is only dependent on the equipment's age and a second function, independent of its age that incorporates the effects of the system state. Since the collected information during an observation is imperfect, the condition of the system cannot be precisely determined. The Bayes' rule is used to determine the probability of being in a certain state at each observation point. We assume that the equipment's unobservable state transition follows a Hidden Markov Model. Two numerical examples with unobservable condition are studied and the expected remaining life is analyzed.

Keywords: Condition Based Maintenance, Imperfect Information, Expected Remaining Life, Hidden Markov Model, Proportional Hazards Model.

Nomenclature

\[ T \]: Failure time
\[ Z(t), X(t) \]: Equipment's condition at time \( t \)
\[ Z_k, X_k \]: Equipment's condition at \( k \)-th observation point
\[ p_{ij} \]: Probability of going from state \( i \) to state \( j \) knowing that equipment has not failed
\[ q_{j\theta} \]: Probability of getting information value \( \theta \) while in state \( j \)
\[ \Delta \]: Observation period
\[ h(t, X_k) \]: PHM hazard function
\[ h_\psi(.) \]: Baseline hazard function
\[ \psi(.) \]: State effect function
\[ R(k, X_k, t) \]: Conditional reliability at period \( k \) while the state is \( X_k \)
\[ \tau(k, X_k, t) \]: Conditional mean sojourn time at period \( k \) while the state is \( X_k \)
\[ \pi^k \]: Conditional probability distribution of equipment state at period \( k \)
\[ \pi_i^k \]: Probability of being in state \( i \) at period \( k \)
\[ \overline{R}(k, \pi^k, t) \]: Expected conditional reliability of equipment
\[ e(k,i) \]: Remaining life at period \( k \) when the state is \( i \)
\[ \overline{e}(k, \pi^k) \]: Expected remaining life at period \( k \) when conditional probability distribution of equipment is \( \pi^k \)

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1 Introduction

Condition based maintenance (CBM) is based on observing and collecting information concerning the condition of equipment, in order to prevent its failure and to determine the optimal maintenance actions. When equipment is subjected to CBM, data concerning one or more indicators of degradation is collected periodically. The information obtained from this data is used to establish a diagnosis of equipment condition and a prognosis for its future performance. Two measures of this performance are the failure rate or the hazard function, and the remaining life. These two measures are calculated from the reliability function.

In reliability analysis, two reliability functions are of interest, the unconditional reliability function given by the probability $P(T>t)$, which is the probability that the failure time $T$ of equipment that has not yet been put into operation is bigger than time $t$, and the conditional reliability function calculated by $P(T>t|T>\tau)$, which is the probability that the time to failure is bigger than $t$, knowing that the equipment has already survived until time $\tau$, where $\tau < t$. In the latter, the expected remaining life is given by $E(T-\tau|T>\tau)$, which is equal to $\int_\tau^\infty (T-t)P(T>t|T>\tau)dt$ , and the hazard function is given by $P(t < T < t + \Delta t | T > t)$ . In reliability analysis, it is assumed that all the equipments are used in the same environment and under the same conditions. This assumption allows the calculation of the remaining life and the hazard function prior to actual use of the equipment. In real life, the environment in which equipment is used and the conditions of utilization affect the process of degradation and consequently the remaining life and the failure rate. Taking these factors into consideration improve the prognosis for future performance.

Many researchers have proposed different reliability models that incorporate the information gathered periodically about equipment condition. These models are used to calculate adjusted hazard function and remaining life, taking into consideration the condition of the equipment. One of these models is the Proportional Hazards Model (PHM), proposed by Cox [1]. This model has been widely used in the medical field (Crowley and Hu. [2]; Leemis [3]), then by Jardine et al.[4][5][6] in the field of CBM. This model has the advantage of improving maintenance policies since it is based on a more accurate estimate of the hazard function and the remaining life, and consequently on the optimal replacement decision (Banjevic and Jardin [7]).

To our knowledge, in all previous applications of the PHM, it was assumed that the information was perfect, i.e. it revealed directly the state of the equipment. Realistically, the information may contain noise due to errors of measurement, interpretations, accuracy of measurement instruments, etc. This paper considers the case of imperfect information, i.e. the state of the equipment is unknown and the gathered information is used to calculate the probability of being in a certain state. The PHM introduced by Cox [1] is thus modified to account for previously mentioned uncertainties, and to calculate the conditional reliability and the remaining life based on this modification.

The paper is organized into four sections. Section 2 presents a literature review of the principal models used in the evaluation of the remaining life. Section 3 introduces the proposed model that includes the imperfect information. In section 4, two numerical examples are presented. Conclusions are presented in section 5.
2 Literature review

Many researchers have studied the mathematical structure of remaining life based on reliability analysis without considering information concerning actual use and the state of the equipment. Tang et al. [8] regarded the remaining life as a random variable and studied it asymptotic behaviour when the reliability function is represented by various discrete and continuous distribution functions. Lim and Park [9] studied the monotonic behaviour of the remaining life. They tested the null hypothesis that the remaining life is not monotone against the alternative hypothesis that it is indeed monotone. Siddiqui and Caglar [10] treated the remaining life as a random variable and gave a representation of its distribution function. When this distribution is Gamma or Weibull, the authors calculate the mean and the variance of the variable. Bradley and Gupta [11] also studied the asymptotic behaviour of the remaining life.

Wang and Zhang [12] modelled the remaining life while information is gathered periodically. They called the remaining life “the residual life”, \( e(t, z(t)) \), and defined it as the expected time interval between the last inspection, when the most recent information was gathered, and the expected time of failure, given that no maintenance action is taken in this interval. The residual life \( e(t, z(t)) \) is thus equal to \( E((T-t)|T>t, z(t)) \), where \( z(t) \) is the information gathered at time \( t \). Kumar and Westberg [13] calculated the residual life when only the most recent information is available. Wang and Christer [14] used a recursive filter in order to calculate the residual life, and added to the existing residual life models the possibility of including all past information. The proportional residual life \( e(t|z) \) is introduced by Maguluri and Zhang [15]. They are inspired by the PHM and calculated the proportional residual life using the equation \( e(t|z) = \exp(-\beta z)e_0(t) \), where \( e_0(t) = E(T-\tau |T>\tau) \) is the remaining life calculated without including the gathered information, \( z \) is the vector of information and \( \beta \) is the vector of coefficients. Sen [16] calculated what was named “the conditional residual life” given by

\[
e(t|z) = \hat{\lambda}(t) e^{\beta z(t)} d\mu, \quad \text{where} \quad \hat{\lambda}(t) = \int_{0}^{\infty} \hat{\lambda}_0(u) du, \quad \hat{\lambda}_0(t) \text{ is the hazard function},
\]

and \( \hat{\lambda}(t|z) = \hat{\lambda}_0(t) e^{\beta z} \) is the proportional hazard function. Benjavic and Jardine [7] calculated the joint distribution of time to failure and the equipment state \( z(t) \) at time \( t \), and the probability of transition between states \( \Lambda_j(t) = P(T>t, z(t) = j | T>\tau, z(\tau) = i), \quad \tau \leq t \). The conditional reliability is thus given by the equation \( R(t|\tau, z(\tau)) = \sum_j \Lambda_j(\tau, t), \quad \tau \leq t \), and the remaining life is \( e(t, z(t)) = \int_{\tau}^{\infty} R(\tau|t, z(t)) d\tau \). All of these models assume that the information directly reveals the equipment state \( z(t) \), i.e. the information is perfect. Moreover, some of them include only the most recently collected information. In the next section, the remaining life is modelled while the information is imperfect, i.e. the equipment state is unknown. The model takes into consideration all previous information.
3 Model assumptions

We consider the PHM proposed by Cox [1], and we assume that the information that we collect is imperfect, thus the state of the equipment is unknown. The equipment is described as follows:

- It has a finite and known number of degradation states \( S = \{1, 2, \ldots, N\} \).
- The transition from one state to another follows a Hidden Markov Model (HMM) where the state is unobservable. The transition matrix \( P \) is known or can be calculated. \( p_{ij} \) is the probability of going from state \( i \) to state \( j \) knowing that the equipment has not failed.
- The gathered information is stochastically related to equipment state. The information matrix \( Q \) is known and is equal to \( Q = [q_{j\theta}] \), \( j \in S, \ \theta \in \Theta \). \( q_{j\theta} \) is the probability of getting information of value \( \theta \) while in state \( j \).
- The observations are collected periodically at fixed intervals \( \Delta \). The probability of being in a certain state, the conditional reliability, the failure rate and the remaining life are calculated at these intervals.
- The only observable state is the state of failure that can happen at any time and is know immediately.

Figure 1 depicts the process of degradation and the transition from one state to another, and from working states to the state of failure. State 1 is the best (new equipment). State \( N \) is the worst. It should be noted that failure can happen at any time and while the equipment is in any state. \( \{X(t) = 1, 2, \ldots, N\} \) is a discrete homogeneous Markov process, where \( p_{ij} = \Pr(X(t + \Delta) = j \mid X(t) = i, T > t + \Delta), \ t = 0, \Delta, 2\Delta, \ldots, T \) is a random variable showing the failure time and \( f_i \) is the probability of going from state \( i \) to the state of failure. The circles represent the states.

At time \( t = 0 \), the equipment is always in state 1. At fixed interval \( \Delta \), i.e. at \( t = \Delta, 2\Delta, \ldots \), an observation is collected, the value of this observation is \( \theta \in \Theta = \{1, 2, \ldots, M\} \). For every observation \( \theta \), there is a probability \( q_{j\theta} \) of being in state \( j \). The transition matrix \( P = [p_{ij}] \), \( i, j \in S \) is an upper triangular matrix, i.e. \( p_{ij} = 0 \) for \( j < i \) since the state cannot improve by itself, which is the case in most practical problems.

Failure of the equipment which is immediately obvious and causes the system to cease functioning can happen any time. This failure characteristic is referred to as obvious failure as opposed to silent failure, which is not immediately observed and must be discovered through various actions (Hontelez et al. [17] and Maillart [18]). If failure happens, it is immediately recognized, and the only possible action is “failure replacement”. At any observation point, two actions are possible: “preventive replacement” or “do-nothing”. The replacement action renews the equipment and returns it to state 1.

In this research, it is assumed that all model parameters are known. The objective is to derive the conditional reliability and the remaining life when the information is imperfect, i.e. when the equipment state is unknown.
4 Modelling the remaining life

In this model, the hazard function \( h(t, X_k) \) follows the PHM and is represented by the equation:

\[
h(t, X_k) = h_0(t) \psi(X_k), \quad k = 0, 1, 2, \ldots
\]

(1)

where \( h_0() \) is the hazard function of a Weibull distribution and represents the aging process, and \( \psi() \) is a function of the equipment state (its condition). The most used function is usually exponential in the form \( \psi(X_k) = \exp(\gamma X_k) \). This means that the failure rate depends on the equipment’s age and condition.

Since the observations are gathered at fixed intervals \( t = 0, \Delta, 2\Delta, \ldots \), the notation \( X_k = X(k\Delta) = X(t) \) is used, and the state is assumed to be constant during each interval. Each change of state is assumed to take place at the end of the interval, exactly before the observation point.

In the initial PHM with known states, the conditional reliability is given as follows:

\[
R(k, X_k, t) = P(T > k\Delta + t \mid T > k\Delta, X_1, X_2, \ldots, X_k), 0 < t \leq \Delta
\]

\[
= P(T > k\Delta + t \mid T > k\Delta, X_k), 0 < t \leq \Delta \tag{2}
\]

\[
= \exp \left( -\psi(X_k) \int_{k\Delta}^{k\Delta+t} h_0(s) ds \right), 0 < t \leq \Delta
\]

This conditional reliability indicates the probability of survival at time \( t \), \( 0 < t \leq \Delta \), after \( k\Delta \), knowing that the failure has not happened until time \( k\Delta \), and the states of the equipment have been \( X_1, X_2, \ldots, X_k \), at \( \Delta, 2\Delta, \ldots k\Delta \). \( T \) is the random variable indicating the time to failure. Also the conditional mean sojourn time, if no action is performed before time \( t \), while the equipment is in state \( X_k \) at interval \( k\Delta \), is (Makis and Jardine [19]):

\[
\tau(k, X_k, t) = \int_{0}^{t} R(k, X_k, t) dt, 0 < t \leq \Delta \tag{3}
\]

Equations (2) and (3) are not valid for \( t \geq \Delta \), since \( X_k \) may change.

The conditional reliability at \( (k, X_k) \), i.e. at the \( k \)-th observation point while the state is \( X_k \) and for \( t > \Delta \) is calculated by the following equation:

\[
R(k, X_k, t) = P(T > k\Delta + t \mid T > k\Delta, X_1, X_2, \ldots, X_k), t > \Delta
\]

\[
= P(T > k\Delta + t \mid T > k\Delta, X_k), t > \Delta
\]

\[
= \sum_{j=1}^{N} R(k, X_k, \Delta) p_{X_j, j} \Pr \left( T > (k + 1)\Delta + (t - \Delta) \mid T > (k + 1)\Delta, X_{k+1} = j, t > \Delta \right) \tag{4}
\]

\[
= \sum_{j=1}^{N} \sum_{i} R(k, X_k, \Delta) p_{X_i, j} \Pr \left( T > k\Delta + t \mid T > (k + 1)\Delta, X_{k+1} = j \right), t > \Delta
\]
“1” is the probability of survival until \( \Delta \), “2” is the probability of transition from state \( X_k \) to state \( j \), and “3” is the probability of survival until \( k\Delta + t \) while at the \( k+1 \)st observation point the state is \( j \). Equation (4) can be written as follows:

\[
R(k, X_k, t) = R(k, X_k, \Delta) \sum_{j=1}^{N} p_{X_k,j} R(k+1, j, (t-\Delta)), t > \Delta \\
(4')
\]

Considering equations (2) and (4), the conditional reliability at the \( k \)th observation point, while the state is \( X_k = i \) and no action is taken is:

\[
R(k, i, t) = \begin{cases} 
\exp \left( -\psi(i) \int_{k\Delta}^{k\Delta + t} h_0(s) ds \right) & 0 < t \leq \Delta \\
R(k, i, \Delta) \sum_{j=1}^{N} p_{ij} R(k+1, j, (t-\Delta)) & t > \Delta 
\end{cases}
(5)
\]

In this paper, the state is unobservable, but an indicator is observed and its value \( \theta \) is recorded. A new state space and transition rule are introduced. They include all the observations history from the last renewal point, and provide a methodology to deal with unobservable states by calculating the conditional probability \( \pi^k_i \) of being in state \( i \) at time \( k \). \( \pi^k_i \) is be the conditional probability distribution of the equipment state at period \( k \), and is defined as follows:

\[
\pi^k = \left\{ \pi^k_i, \quad 0 \leq \pi^k_i \leq 1 \text{ for } i = 1,...,N, \sum_{i=1}^{N} \pi^k_i = 1 \right\}, \quad k = 0,1,2,...
(6)
\]

and

\[
\pi^0_i = \begin{cases} 
1 & i = 1 \\
0 & 1 < i \leq N 
\end{cases}
(7)
\]

Equation 7 means that new or renewed equipment will always begin in state 1.

After an observation \( \theta \) is collected, the prior conditional probability is updated to \( \pi^k_j(\theta) \). By using Bayes’ formula, and knowing that the observation \( \theta \) has occurred at the \( k+1 \) observation point, \( \pi^k_j(\theta) \) is given as follows:

\[
\pi^{k+1}_j(\theta) = \frac{\sum_{i=1}^{N} \pi^k_i p_{ij} q_{ij\theta}}{\sum_{i=1}^{N} \sum_{l=1}^{N} \pi^k_i p_{il} q_{l\theta}}, \quad j = 1,...,N
(8)
\]

The updated conditional distribution carries the history of all the observations and actions from the last replacement point. After any preventive or failure replacement, the periods’ counter will be reset to zero and the conditional probability distribution of the equipment state will be set to \( \pi^0 \).

In this case of unobservable states, we define \( \overline{R}(k, \pi^k, t) \) as the expected conditional reliability of the equipment at the \( k \)-th observation point, while the state conditional probability distribution is \( \pi^k \). It is calculated as follows:

\[
\overline{R}(k, \pi^k, t) = \Pr(T > k\Delta + t \mid T > k\Delta, (k, \pi^k)) = \sum_{i=1}^{N} R(k, i, t) \pi^k_i
(9)
\]
By substituting equation (5) into equation (9) we get:

\[
\overline{R}(k, \pi^k, t) = \begin{cases} 
\sum_{i=1}^{N} \pi_i^k \exp \left( -\psi(i) \int_{k\Delta}^{k\Delta+t} h_0(s) \, ds \right) & 0 < t \leq \Delta \\
\sum_{i=1}^{N} \pi_i^k R(k, i, \Delta) \sum_{j=1}^{N} p_{ij} \times R\left(k+1, j, (t-\Delta)\right) & t > \Delta
\end{cases}
\]

(10)

When the states are observable, the remaining life is given by Banjevic and Jardine [7] as follows:

\[
e(k, i) = E(T - k\Delta | T > k\Delta, X_k = i) = \int_0^\infty R(k, i, k\Delta + t) \, dt = \int_{k\Delta}^\infty R(k, i, t) \, dt
\]

(11)

In this case of unobservable states, we define the expected remaining life, \(\overline{e}(k, \pi^k)\) calculated at the \(k\)th observation point, and while the state conditional probability is \(\pi^k\) as follows:

\[
\overline{e}(k, \pi^k) = E\left(T - k\Delta | (k, \pi^k)\right) = \sum_{i=1}^{N} \pi_i^k e(k, i) = \sum_{i=1}^{N} \pi_i^k \int_{k\Delta}^\infty R(k, i, t) \, dt
\]

\[
\overline{e}(k, \pi^k) = \sum_{i=1}^{N} \int_{k\Delta}^\infty \pi_i^k R(k, i, t) \, dt = \int_{k\Delta}^\infty \left( \sum_{i=1}^{N} \pi_i^k R(k, i, t) \right) \, dt
\]

(12)

The steps for calculating the expected remaining life at each observation point \(k \geq 1\), where the observation obtained is \(\theta\), are as follows:

- Update the conditional probability distribution \(\pi^k\), at period \(k\) by using equations (6) to (8).
- Calculate expected conditional reliability of the equipment \(\overline{R}(k, \pi^k, t)\), at the \(k\)-th observation point by using equation (10).
- Calculate expected remaining life \(\overline{e}(k, \pi^k)\), by applying equation (12).

5 Numerical example

The following example is adopted from Ghasemi et al. [20]. The hazard function \(h_0(t)\), representing the aging process follows a Weibull distribution, and the equipment condition, \(\psi(X_i)\) is given in an exponential form as follows:
\[
h(k, t) = \frac{\beta^\alpha t^{\alpha-1}}{\alpha^\beta}, t \geq 0, \alpha = 1, \beta = 2
\]

\[
\psi(X_i) = e^{0.5(X_i-1)}.
\]

For \( \Delta = 1 \), the equipment’s hazard function and its conditional reliability are thus given as follows:

\[
h(t, X_r) = 2te^{0.5(X_r-1)}
\]

\[
R(k, X_r, t) = \exp[-((t^2 + 2tk)e^{0.5(X_r-1)})]
\]

In addition to the observable failure state, the equipment can be in any of two unobservable states \{1, 2\}. 1 is the new or as new state.

The transition matrix \( P \) is given as follows:

\[
P = \begin{bmatrix}
0.4 & 0.6 \\
0 & 1
\end{bmatrix}
\]

The observation values \( \theta \) are classified in three nominal conditions; excellent, normal, and degraded. The probabilities of observing one of these three conditions while in state 1 or 2 are given by the information matrix \( Q \) as follows:

\[
Q = \begin{bmatrix}
0.6 & 0.3 & 0.1 \\
0.2 & 0.4 & 0.4
\end{bmatrix}
\]

Based on the developed model, the expected remaining life at different observation points \( k = 0,1,2,... \) is shown in table 1. The value of the expected remaining life at \( k = 0 \) is not applicable (N/A) since we assume new equipment is always in state 1. According to the calculations, this equipment has an expected remaining life equal to 0.8013.

We note that in a real situation, after the collection of an observation \( \theta \), the probability of being in state \( i=1,2 \) is calculated. This probability is used in equation (12) to obtain the expected remaining life. Since in this example, the values of observations are not available, the expected remaining life is calculated for several possible values of \( \pi_2^k \), the probability of being in state 2 while at the kth observation point (see table 2).

Figure 2 shows the values of the expected remaining life at different values of \( \pi_2^k \). As expected, the value of the expected remaining life decreases as the probability of being in state 2 increases.

To explain the relationship between the optimal replacement policy that was obtained in (Ghasemi et al. [20]), and the expected remaining life, we recall that in that paper the cost of preventive replacement is \( C = 5 \), while the cost of replacement after failure increases by \( K = 2 \). The optimal replacement criterion is given as a function of the expected reliability and the expected sojourn as follows:

\[
T_{g^*} = \inf \left\{ k \geq 0 ; \ 2 \times \left[ 1 - \bar{R}(k, \pi^k, \Delta) \right] \geq 8.1704 \times \bar{\tau}(k, \pi^k, \Delta) \right\},
\]

where \( g^* = 8.1704 \) is the long run average cost of replacement. Figure 3 shows the decision criterion for this example. The straight line indicates the threshold value of \( g^*/K \). It can be seen that independent of the value of \( \pi_2^k \), it is never cost optimal to replace after the first interval. Similarly, after the second interval, the equipment should optimally be replaced regardless of the value of \( \pi_2^k \). This means that after two periods of
utilization, the equipment should be replaced regardless of its state. This decision takes into consideration the costs of replacement as well as the value of the expected conditional reliability. Another way of decision making would be to consider the value of the expected remaining life. For example, in figure 3, the decision based on cost and the value of the conditional reliability is to never replace at \( k=1 \), i.e. after the first period. However, by considering the value of the expected remaining life given in table 2, which depends on the observation collected at the end of this period, the practitioner may decide differently. For example, if the updated \( \pi^k \) is equal to 0.9, then the expected remaining life is equal to 0.001901, which is very small and smaller than the length of an interval. The decision maker may then decide to replace the equipment although it is not the cost optimal decision. This decision is not based on cost considerations.

To further explain this criterion, we assume that \( C = 5 \) and \( K = 4 \), which will result in \( g^* = 10.17 \) and the threshold line will shift to \( g^*/K = 2.54 \) as shown in Figure 4. This shift means that if at the first observation point the calculated amount of the \( \pi^1 \) is larger than or equal to 0.3, the equipment should be replaced; otherwise it should be replaced at the next observation point. From table 2, at \( \pi^1 = 0.3 \), the expected remaining life is 0.006167. This gives another indication for the decision maker to whether he or she should replace the equipment.

Another example is shown in figure 5 and 6. In this example, the equipment may be in one of three working states \( i=1, 2, 3 \). The transition matrix is \( P = \begin{bmatrix} 0.9 & 0.1 & 0.0 \\ 0.0 & 0.9 & 0.1 \\ 0.0 & 0.0 & 1.0 \end{bmatrix} \), the information matrix is \( Q = \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.2 & 0.3 & 0.5 \\ 0.0 & 0.0 & 1.0 \end{bmatrix} \), the Weibull distribution parameters are \( \alpha = 3, \beta = 3, \Delta = 1, C = 5, K = 2 \). As for the previous example, based on the optimality condition that minimizes the cost of replacement in a renewal cycle, the decision is not to replace after the first interval and to replace after the second interval if the corresponding point of \( (\pi^1, \pi^2) \) on the surface indicating \( k=2 \) is above the replacement threshold surface in figure 5. Another criterion for decision making is the value of the remaining life given in figure 6.

6 Conclusion

While in most published papers, the remaining life is calculated when the states are observable, in this paper, the expected remaining life is modelled and calculated for equipment with unobservable states and obvious failure. The model is based on the PHM with time to failure following a Weibull distribution and equipment condition represented by an exponential function. The expected conditional reliability is derived from the PHM and used to calculate the expected remaining life. Two examples are presented. The optimal replacement policy and the expected remaining life are calculated at several possible state probabilities. It has been shown that the expected remaining life can be
used as a decision tool, in particular when the cost elements of preventive replacement are unknown.

References


Figures

Figure 1: The process of degradation and failure

Figure 2: The expected remaining life at different values of $\pi_2^k$ and $k = 1$
Figure 3: Decision Criterion for $\Delta = 1$, $K=2$ and $C=5$

Figure 4: Decision Criterion for $\Delta = 1$, $K=4$ and $C=5$
Figure 5: Decision criterion for equipment with three working states

Figure 6: The expected remaining life at different values of $\pi_1^k$, $\pi_2^k$ and $k = 1$
### Tables

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**Table 1:** The expected remaining life at different observation points

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**Table 2:** The expected remaining life for different possible $\pi_2^k$