Waves Beneath the Sea[1]

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Abstract: Most of us are familiar with the waves that bob on the surface of the ocean. Some of us can even tell stories of the "stomach" memory of such waves! But most people don't know that the ocean also has waves that bob deep below the surface. Besides being interesting in their own right, these so-called "internal waves" provide a convenient way to explain some key concepts in Physics. I'll outline such topics here, along with an experiment that can be done by students.

Introduction

Beneath the sea surface, unseen by human eyes, lie "internal" waves. They exist because the deep waters of the ocean are denser than the surface waters. [*Your class might be able to tell you why the surface waters are buoyant: they are heated by the sun at low latitudes, and freshened by melted ice at high latitudes.*] If a parcel of deep (heavy) water were to be pulled up towards the surface, gravity would force it back downwards. It's the same thing that would happen if you raised a parcel of sea water into the air; it would fall when released. The reverse also holds. The buoyancy of surface waters makes them return to the surface if they are momentarily pushed downwards. [*If students find it odd that gravity can make things rise, get them to think of a (buoyant) piece of wood floating on the ocean surface, momentarily pushed downwards.*]

Restoring Force

This rising of depressed water, and sinking of uplifted water, is an expression of a "restoring force".

> **Editorial Aside**: *The phrase "restoring force" may be familiar to your students from their study of Physics. If not, then they are missing out on an important, and fascinating, part of Physics. All oscillations are caused by restoring forces. It might be useful to quiz students on this topic, by asking them to discuss the restoring forces for such oscillators as pendulums, drum skins, vocal cords, a car with poor shock absorbers, and, yes, even waves!*

It is this restoring force, acting on vertical variations of density, that causes internal waves. The concept is the same as that for surface waves; in each case there are two layers of fluid with a wavy interface between them, as illustrated in Figure 1.

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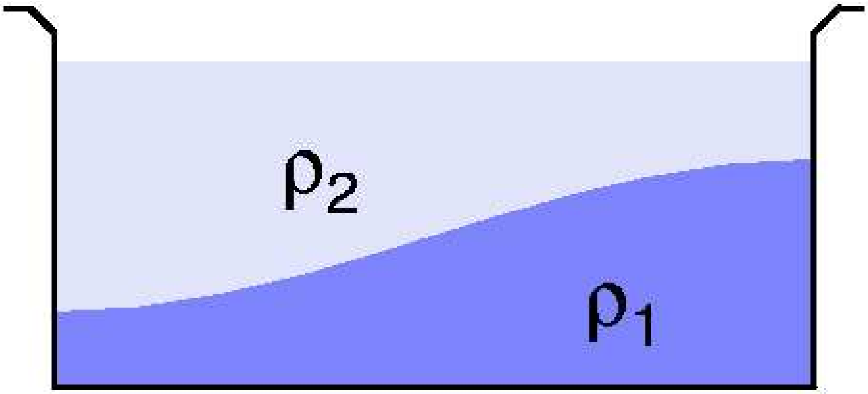

Figure 1. Side view of a beaker with two "layers" within it, one of density ρ_1 and the other of density ρ_2. [*The Greek letter* ρ, *pronounced "rho" is commonly used for density, in all branches of Fluid Mechanics, e.g. Oceanography, Meteorology, etc. I mention this detail because I've found that many students don't know the Greek alphabet, and that this makes it difficult for them to understand lectures or textbooks, which are awash in Greek squiggles!*]

For surface waves, the upper fluid is air, the lower fluid water, and the difference in density between these two, $\rho_1 - \rho_2$ in the notation illustrated above, is about **1000 kg/m^3**. The density variations within the ocean are comparatively small, and as a consequence the restoring force for internal waves is weak compared with that for surface waves.

It is for this reason that internal waves move very slowly within the ocean. But how slowly, exactly? To answer this, we need some Mathematics to answer this.

The typical density of surface seawater is **ρ_1=1024 kg/m^3** while the deep density is typically of value **ρ_2=1025 kg/m^3**. (These vary of course; here I'm giving reasonable values for the purpose of illustration.) What's most important for internal waves is the difference in density, and we denote that by putting a Greek upper-case delta before the density symbol:

$$\Delta\rho = \rho_1 - \rho_2 \qquad \text{... equation 1}$$

It is also convenient to denote the mean density, and for that it's common to use a subscript zero:

$$\rho_0 = (\rho_1 + \rho_2) / 2 \qquad \text{... equation 2}$$

Thus, the relative difference in density is $\Delta\rho/\rho_0$. It would be a good exercise for students to insert the above numbers into equations 1 and 2, and to state whether the relative density difference is a big number or a small one. [*It's small!*] Then, you could ask them whether the same value would be achieved if we just used either ρ_1 or ρ_2 instead of ρ_0 in the formula for relative density difference. [*It would, and for that reason, in calculations we worry about the value of* $\Delta\rho$ *but we just pick a roughly representative number for the* ρ_0 *value. Perhaps this is a good example of a broader lesson, on significant digits, on deciding when to stop copying digits out of a calculator, etc?*]

But why am I interested in $\Delta\rho/\rho_0$ and whether it is small or large? The answer is related to the effective gravity, or the "buoyancy." Let g denote the acceleration due to gravity. [Y*our class should know that* **g** *is roughly* ***10 m/s²***. *If not, they can figure this out by dropping things off the roof of the school, as I did when I was a student, if the lawyers let kids go on the roof anymore!*] It turns out that the *effective* gravity felt by internal waves is given by the formula

$$g' = g\,\Delta\rho / \rho_0 \qquad \text{... equation 3}$$

where the symbol g' is commonly called the "reduced" gravity. [*At this stage it would make sense to suggest that students calculate* g' *using the above density values, to verify that it is, in fact, "reduced."*]

Now, let's return to Figure 1. Notice that I've drawn the interface between the deep, heavy water and the surface, light water with a wiggled line. I think most students, when asked what will happen next, will say that the lower water will slosh from the right-hand side of the tank, over towards the left-hand side. (At the same time, the surface water will slosh in the opposite direction.) Most students will also intuit that this water will bounce off the side of the tank, and slosh backwards to the right again. In other words, they will realise that a wave will slosh back and forth across the tank. [*When I say "students will realise," I mean it literally. I've found it helpful, in terms of motivation, to ask students to make a prediction of what may happen, based on their intuition. Then, when the teacher explains the Mathematics, the students will be paying close attention, to see whether the Mathematics will agree with what they have proposed. Predict, then test: the classroom activity becomes a model of the scientific method.*]

Speaking of Mathematics, now is the time for me to talk about one of the prime properties of internal waves: their speed. But, before I do that, I ask the reader to write two numbers on a piece of paper: first, the typical speed of ocean surface waves (in m/s please; walking speed is about 1 m/s) and, second, the speed of ocean internal waves. Sure, you may have to guess at the latter, but why not have some fun while you read? :-)

Propagation Speed

According to a theory that is beyond high-school (since it involves the solution of partial differential equations, often not taught until the third year of university in these days of Mathematics fear) the speed of surface water waves, V, is given by the formula

$$V = (gH)^{1/2} \qquad \text{... equation 4}$$

in which H denotes the water depth. For example, the water near local beaches is roughly **1m** to **10m** deep, so that the speed of waves approaching our beaches is, substituting into the above equation, roughly **3m/s** to **10m/s**. For a point of reference, your students can run at **3 m/s,** but only an Olympic athlete can run at **10m/s,** and then only for a few tens of seconds. [*There's no need to tell students this; make them tell you!*]

For internal waves, the formula remains the same, except that g' is used instead of g in the square root. [*Actually, there is another change. The* ***H*** *used in the internal-wave formula is the geometric mean of the thicknesses of the upper and lower layers.*] If you look back to the example density values given above, you'll see immediately that internal waves must move slowly compared with surface waves. A typical value is $g'=0.01\ \mathrm{m/s^2}$, and therefore the speed will be reduced by a factor of about **1/30** (since **30** is roughly equal to $\sqrt{1000}$).

Consider the surface/internal waves of the waters near Nova Scotia. The rough depth, for distances from a kilometre to a few hundred kilometres from shore is H**=100m**. So, the surface waves move at a speed of **30m/s** while the internal waves move at a comparatively sluggish **1m/s**. [*To get an idea of the speeds, ask students how fast cars go on the highway, and also how fast folks walk in a wedding procession.*] That's a dramatic difference in speed, and one that is very important to Oceanography, as the examples in the next section illustrate.

Applications

Dead Water

In the early years of the 1900s, ocean science was just beginning to make linkages with Physics and Mathematics. An early application was an explanation of something that mariners had complained about for generations: "dead" water. By this phrase, they meant the fact that ships entering fjords often found that their speed was dramatically reduced. No matter what the wind conditions, and no matter what the tide, ships found it nearly impossible to make normal headway when they entered a fjord. The ship "died" in the water, literally!

The problem is illustrated in the picture below.

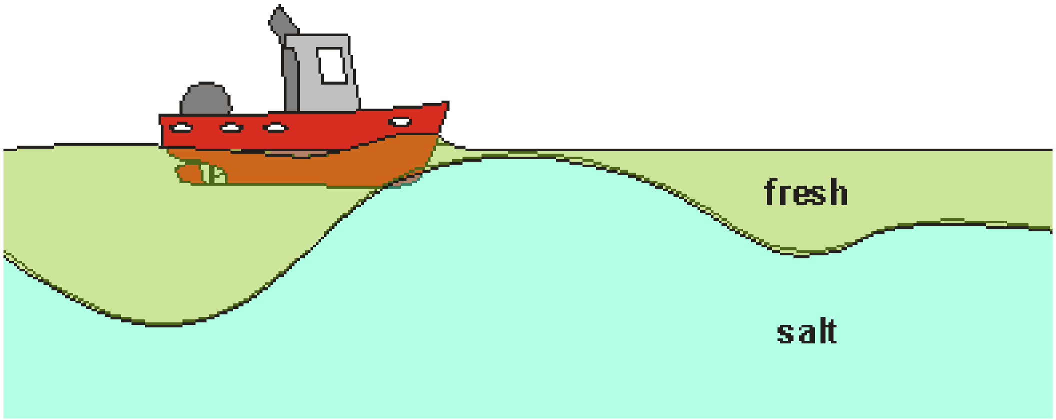

Figure 2. Side view illustrating "dead water."

In a fjord, the surface waters tend to be very fresh, due to land runoff, while the deep waters are salty since they derive from the open ocean. This leads to a two-layer system, as illustrated in Figure 2. (In the open ocean, winds mix the fresh water into the salt water, but that mixing is inhibited in the fjord because the land shelters the water from the wind, and the coasts prevent horizontal stirring of the water.)

Ask students to look at the bow of the ship. They should notice that the bow is making a "bow wave" in the ocean surface. At a typical water depth of H=**10m** to **100m**, the speed of this wave is **10m/s** to **30m/s**. Since these wave speeds are much faster than the typical ship speed, the wave won't hold the ship back. [*Some of your students might sail; if so, ask them how fast their boats go. They will answer in "knot" units. A knot is about* ***0.5 m/s****; again, this conversion is something that you could ask students about, since they know how many metres in a mile, and how many seconds in an hour.*]

However, there is also a "bow wave" within the water, which is caused if the boat is deep enough to touch the lower layer. The ship has to push a great deal of water in front of it, and that water cannot go any faster than internal waves. But, as we've seen, internal waves move slowly. Therefore, the ship (which is "surfing" on this internal wave) is forced to slow down. That's why the ship slows down when it enters the fjord. [*Ask your students if the same thing would happen if the ship did not extend deeply into the water.*]

Since it is hard to see internal waves from the bridge of a ship, mariners were greatly mystified by this "dead water" phenomenon. But, armed with the two pictures above, and the formula for the speed of internal waves, it's easy to see the Physics of the dilemma. As to a solution ... well, there isn't any, really, except lots of power or throwing valuable cargo overboard to float higher in the water!

Seiching

The wave in Figure 1 will bounce off the wall, turn around, and then bounce off the other. With no friction, this would go on forever. This wall-to-wall bouncing is called "seiching", and it occurs for both surface waves and internal waves. For a given wave speed, and a given distance between walls, there is a particular period of these seiche waves. This is called the "resonant period" of the system. The formula for the period is $L / (4V)$ where L is the distance from wall to wall, and V is the speed of the waves, whether they be surface waves or internal waves.

A good example of surface-wave seiching is the sloshing of water in a tub. [*Students might like to do an experiment at home, with measurement of water depth, resonant period, etc.*] In the ocean, a good local example for surface seiching is the tidal resonance in the Bay of Fundy. As for internal-wave resonance, you can tell your students that if they come to Dalhousie, they can take my course, and go to the Bedford Basin and measure this seiching for themselves! [*But, make sure to tell them to take lots of Mathematics courses first. Mathematics is the key to getting beyond the picture-drawing phase.*]

Present-day Research Applications

As it turns out, part of my research programme involves internal waves. Why are oceanographers interested in internal waves? There are many reasons. Military folks are very interested in learning more about internal waves because moving submarines make such waves, just as a surface ship makes surface waves, and therefore submarines can be detected just by looking for internal waves. [*For a connection to a history course, consider the fact that submarines can hide right next to the coast, and so they can launch a missile to a continental target so quickly that defence is impossible. Proximity is the most important factor in missile defence, as students who've read about the Cuban missile crisis could explain.*] Biologists are interested because internal waves bring nutrient-rich water up near the surface, where there is light from the sun, and therefore the waves can increase photosynthesis of microscopic plants in the water. Climate researchers are interested because internal waves can break, in much the same way as surface waves break, and this causes vertical mixing of heat from the surface into the deep waters. Indeed, there is a new theory (presently being tested with measurements near Hawaii) that suggests that internal-wave mixing is an important factor in the whole climate system!

Laboratory Experiment

The situation pictured in Figure 1 is easy to set up in the classroom. Just get an aquarium at the local store (or make one out of plexiglass or glass) and fill it with two layers of fluid. If the fluid is water, you can control the density either by heat or by adding solute. The latter is easier, since heat is difficult to work with ... and it goes away when you leave the experiment for a few minutes!

So, the first choice is how to control the density. Both sugar and salt are fine, but salt is easier to clean up, so I recommend it. If you'd like to make seawater, add **30 g** of salt to **1 litre** of water. That would give a density of **1023 kg/m^3**, while fresh water has a density of about **1000 kg/m^3** at room temperature. This yields $g' = 0.2\text{m/s}^2$ as the effective gravity for the internal waves. More generally, students can calculate the density of water at room temperature, at a salinity of S grams of salt per kilogram (litre, basically) of water, with the formula

$$\rho = 995 + 0.76\, S \qquad \text{... equation 5}$$

from which it is evident that the all-important density difference ratio is given by

$$\Delta\rho \,/\, \rho_0 = 8 \times 10^{-4}\, S \qquad \text{... equation 6}$$

to within a few percent, if the top layer is fresh and the bottom layer has salinity S measured in grams of salt per kilogram (roughly, per litre) of water. For reference, the typical salinity of seawater is S=30 to 35 grams of salt per kilogram of seawater. (This may be written as 30 to 35 PPT in some books, or 30 to 35 PSU in others.)

Let's say the tank is **0.5m** wide and **0.2m** high. [*I'll stick to metres, not centimetres, since* *otherwise it's too easy to get mixed up in the formula for speed. As general advice, permit centimetres in rough discussion but insist on metres in calculations!*] Therefore, if the tank is filled to a depth of **0.1m** we get a wave speed of roughly **0.1 m/s**; if the tank is **0.5 m** wide then the wave will take roughly **5 s** to get from side to side. This is slow enough to measure easily, and a good hint on the visualisation is to put some food colouring into the salt water when you're making it.

A good experiment would be to measure the seiche period, i.e. the time it takes for an internal wave to go from one side of the tank to the other and then back. Given all of the above, the prediction is that this period is

$$\tau = \boldsymbol{L} / (\mathbf{4}\ [\boldsymbol{g}\ \boldsymbol{H}\ \Delta\rho/\rho_0]^{1/2}) \qquad \text{... equation 7}$$

where I've used the Greek letter "tau" to indicate the period, as is conventional in Physics.

To test this formula, it would make sense to undertake a suite of experiments, perhaps with different water densities and a fixed geometry, in which case the formula could be tested by plotting the measured period versus the square root of the salinity in the lower layer. The result should be a straight line, and the slope should be given by the formula

$$\text{slope} = \boldsymbol{L} / (\mathbf{4}\ [8\times10^{-4}\ \boldsymbol{g}\ \boldsymbol{H}\ \boldsymbol{S}]^{1/2}) \qquad \text{... equation 8}$$

Another test might be to use different water depths, or different tank widths.

These sorts of experiments are ideal for group work. One student could be assigned to determine how much salt should be added to a bucket of water to achieve a given salinity. Another student could do the measurements and make up the water. Others could work on filling up the tank.

> **Hint:** pour the bottom water in first. Then put a thin sheet of styrofoam on the surface of the water, and gently pour the surface (fresh) water onto the styrofoam. Water will flow outwards, off the edges of the styrofoam, and that prevents the mixing that would occur if you just poured the water. Once the filling is complete, gently remove the styrofoam, being careful not to let it dip back into the water since that could mix up the carefully-prepared layers. Also, either the salty water or the fresh water should have food colouring added ... although depending on the lighting conditions, it's often easy to see the difference in salinity even without adding dye.

Waves can be set up by slowly tilting the tank a few degrees and then slowly lowering the lifted edge back to the table. (This may require a strong student if your tank is large.) After a couple of tries, it is easy to see how to do this, and to understand that the word “slow” above means slow compared with the speed of the waves being set up. Then, let the students time the events, as the wave hits one side and then another. [*Most students have watches, and it would be a great idea* *to have everybody in class do the timing at once. That will let you teach about calculating means and standard errors in measured values*.] Get the students to put all of these points on the graphs they make, and encourage discussion about the scatter in the points about the line.

[You'll gather from the last paragraph that I think this is an ideal laboratory experiment for teaching laboratory/analysis techniques. Many students find these experiments a lot more fun than experiments in other laboratory sessions. If so, it might be helpful to leverage that enthusiasm into general understanding of the techniques of experimental science.]

Summary

Internal waves follow the same Physics as surface waves, but the forcing is weaker so the wave speed is slower. This speed is important in many applications, perhaps most intriguingly in the "dead water" phenomenon.

Internal waves are a good topic in schools because

- they illustrate some important physical concepts (e.g. restoring forces, the nature of gravity acting on fluids, the idea of oscillations, the idea of reflected waves, the idea of resonance, calculation with approximate formulae, etc) and
- young people may be fascinated to learn that there waves hidden below the sea.

Further Reading

For more on this topic, please see the webpage I've set up to outline my presentation at the 2001 Association of Science Teachers meeting in Halifax, NS, Canada. (The AST website, **http://ast.ednet.ns.ca**, has a link.) There, you'll see a clever technique that we've been using at Dalhousie to "see beneath" the surface, and to measure the properties of the internal waves underneath. In the next few years I hope to undertake a series of internal-wave experiments in the St. Lawrence Estuary, funded by the Canadian government and the U.S. Navy, and perhaps I'll report on my results in this journal, if readers express an interest!