

MYOPIC LOSS AVERSION AND MARGIN OF SAFETY: THE RISK OF VALUE INVESTING

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Abstract

This paper examines the risk of value investing from the point of view of a myopic loss-averse investor holding a diversified portfolio and relying on infrequent portfolio rebalancing. This closely resembles purchasing a large portfolio, such as those created by BARRA, and following a buy-and-hold investment strategy. In these circumstances, which portfolio, value or growth, is riskier to a myopic loss-averse investor? To facilitate analysis, a myopic loss ranking and a corresponding statistical procedure are developed and applied to investment-style data provided by BARRA. The paper qualifies the conditions under which value investing is more risky in North American financial markets.

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1 Introduction

This paper examines the risk of value investing from the point of view of a myopic loss-averse investor holding a diversified portfolio and relying on infrequent portfolio rebalancing. This closely resembles purchasing a large portfolio, such as those created by BARRA, and following a buy-and-hold investment strategy. The question then is: in these circumstances, which portfolio, value or growth, is riskier to a myopic loss-averse investor?

What constitutes risk and how it should be measured is the subject of some debate. In regard to value investing, the essential background to this debate is the observed empirical regularity called *value superiority*; that is, the finding that value investment consistently yields superior returns to other investment strategies, notwithstanding the use of different practical means to differentiate value from other stocks. Notable examples documenting value superiority are Bantz (1981), De Bondt and Thaler (1985), Rosenberg, Reid and Lanstein (1985), Sharpe (1988, 1992), Chan, Hamao and Lakonishok (1991), Fama and French (1992, 1996, 1998), and Lakonishok, Schleifer and Vishny (1994).

At the level of practical investment policy, consider the conversation on risk between Eugene Fama and Peter Tanous [see Tanous (1997)]. Fama argues that value stocks are a more risky investment than growth stocks implying that, per unit of price, growth stocks are held at lower expected returns than value stocks (which must have higher expected returns to compensate for the higher risk of holding them). This reasoning is a theoretical inference from the empirical observation of value superiority. The Tanous rejoinder begins by noting that growth stocks are valued at a high multiple of current earnings, to reflect the belief that anticipated earnings are high. “So if a growth stock falters on its anticipated growth path, [its price] declines precipitously because it no longer deserves the [high] multiple that had previously been awarded to it Therefore,

a lot of people think that growth stocks ... are riskier.” In addition, it can be argued that the faltering must initially occur when the price-earnings ratio lies in the upper tail of the overall price-earnings ratio distribution, in view of the valuation with a high multiple. Thus the likelihood that a lower price-earnings ratio can prevail is high, and the downside risk is therefore high. On the contrary, value stocks are low-multiple stocks and so their price-earnings ratio must lie in the lower tail of the overall distribution, and hence the likelihood of an even lower price-earnings ratio must be small. The downside risk must, therefore, be greater for growth stocks than it is for value stocks. Quite clearly these two views of risk differ, the former being the overall *implicit risk* of the higher *expected returns* on value stocks and the latter the greater *downside risk* on the *price* of growth stocks.

Turning now to the empirical finance literature seeking to explain value superiority and the associated risk, this divides itself into three distinct streams. The first is represented by De Bondt and Thaler (1985) which argues that value superiority is a natural consequence of exploiting “contrarian” movements in market prices. Winning (losing) portfolios tend to be losing (winning) portfolios later on as calculated from the cumulated average residuals in a capital asset pricing model (CAPM) for US stocks during the period 1926-82. The earnings-to-price ratio is used to differentiate winners from losers. However, there is no direct attack on the risk of value investment, but a value premium is determined as that part of the return on value stocks which is not accounted for by the broad market portfolio return and its beta.

The second approach, represented by Lakonishok, Schleifer and Vishny (1994), regards value superiority as a consequence of a general preference for growth stocks and the avoidance of value stocks. In theory, this forces down the price of, and raises the return on, value stocks. To test this hypotheses, factors such as the book-to-price ratio, earnings-to-price ratio, and past growth rate in sales are needed to differentiate glamour portfolios from value portfolios. Using US data for the period 1963-90, value superiority

persists across different capitalizations in terms of the average return per year, average return over a five year period, compounded five-year return, and the average annual size-adjusted return over a five year period. The differentiating factors are significant in a regression model for average portfolio return. It is also noted that value strategies are not inherently riskier, based on the positive excess mean return of value strategies during recessions and major market downturns.¹

The third approach, represented by Fama and French (1992, 1996, 1998) and Chen and Zhang (1998), argues that value-investment strategies are inherently more risky than other strategies. Contrary to De Bondt and Thaler (1985), Fama and French find that US data do not support the CAPM in its original form. Using US data for the period 1962-90, Fama and French (1992) note that capitalization and book-to-price ratio (mean return of a value portfolio minus mean return of a growth portfolio) are useful proxies for risk which are not included in the CAPM. An augmented CAPM or multi-factor model is found to explain asset pricing much more accurately; moreover, the risk premium on a portfolio is explained by the broad market portfolio premium, the value premium and a size premium.

Black (1993) and MacKinlay (1995) note that the value premium is sample specific and this leads Fama and French (1998) to examine international experience based on thirteen major financial markets for the period 1975-90 using the augmented CAPM. Chen and Zhang (1998) also examine the behavior of value stocks in six financial markets using the Fama-French (1996) framework. The findings of Fama and French (1998) are confirmed.

Putting these results together, the risk premium on value stocks is measured, in the Fama-French-Chen-Zhang stream, via the coefficients of the augmented CAPM, and in the De Bondt-Thaler stream, as residuals from a standard CAPM. In the Lakonishok-Schleifer-Vishny stream, mean portfolio returns (controlling for investment style and

capitalization) are used, assuming that the premiums on broad market portfolio risk and idiosyncratic risk are inherent parts of any investment style and capitalization.

In seeking to explain value superiority and the associated risk it seems natural to begin with the motives for value investment. Graham (1973, pp. 53-62 and 277-287) argues that a value investor is defensive, seeking a large margin of safety through selective purchase of the stocks of large, prominent, conservatively financed corporations at discounted prices. In economic theory, preferences leading to defensive behavior of this kind were first characterized as loss averse [see Samuelson (1965), Kahneman and Tversky (1979), Fishburn and Kochenberger (1979)]; later, with the addition of frequent mental accounting for near-term losses, the characterization became known as myopic loss aversion or MLA [see e.g. Benartzi and Thaler (1995); Thaler, Tversky, Kahneman and Schwartz (1997)]. A defensive investor has such strong aversion to losses that the possibility of loss must be evaluated frequently, to ensure that long-term losses can be avoided. A loss here is not necessarily an accounting loss but rather an outcome below a certain reference point. Unfortunately, this implies that a shift in the reference point can turn losses into gains and vice-versa [Tversky and Kahneman (1991)].

In this paper, a loss profile and a loss ranking are developed to characterize myopic loss-averse investors and hence the performance of different portfolios. A suitable new statistical procedure is described which allows different portfolios to be ranked in terms of their losses.² Then the statistical procedure is applied to BARRA value and growth portfolios, to determine which portfolio, value or growth, is riskier to a defensive investor. This approach differs from the existing approaches in that it permits the evaluation of risk from a myopic loss point of view, without imposing any further restrictions on the parametric form of the utility function and the underlying return generating process.

The remainder of the paper is organized as follows. In section 2, MLA is formally characterized and a suitable test for it is developed. A detailed description of the BARRA investment-style portfolios is provided in section 3. Application of the pro-

posed test and the empirical results are discussed in section 4. Concluding remarks appear in section 5.

2 Myopic Loss Aversion in a Statistical Framework

2.1 Myopic Loss Profiles and Loss Ordering

Loss aversion can be characterized in different ways. For example, Benartzi and Thaler (1995) describe loss aversion with a utility function having two states: $U(x) = x$ for $x \geq 0$ and $U(x) = 2.5x$ for $x < 0$, where a loss is more heavily weighted than a gain and yields negative utility against the positive utility of a gain. Samuelson (1965), however, describes loss aversion literally as aversion to loss so that a loss is fully accommodated while a gain is totally discounted.

When risky returns are compared and ranked on the probability and extent of losses, the amount by which a return falls below some reference point would seem to be key to how risk perspectives may be evaluated.³

Throughout the paper r_X and r_Y denote portfolio returns generated from corresponding distribution functions F_{r_X} and F_{r_Y} . The corresponding quantile functions for r_X and r_Y , that is, the inverse marginal distributions, are given by Q_{r_X} and Q_{r_Y} .⁴

A loss profile for random return r_X relative to reference point $r \in [a, b]$ is defined as

$$\max(r - r_X, 0). \tag{1}$$

That is, the loss profile for random return r_X takes the value $(r - r_X)$ if $r_X < r$; 0 otherwise. This loss profile characterizes the losses of random return r_X generated from a marginal distribution function F_{r_X} relative to the reference point r .⁵ The family of

loss indices $L_\theta(F_{r_X}; r)$ represents the loss of degree θ ($\theta = 1, 2, \dots$) associated with distribution F_{r_X} relative to reference point r :

$$L_\theta(F_{r_X}; r) \equiv \int_0^{F_{r_X}(r)} [r - F_{r_X}^{-1}(p)]^{\theta-1} dp. \quad (2)$$

By a change of variable, this definition may also be expressed as

$$L_\theta(F_{r_X}; r) = \int_a^r [r - w]^{\theta-1} dF_{r_X}(w). \quad (3)$$

The family of loss indices in the form of $L_\theta(F_{r_X}; r)$ includes some commonly used loss indices, such as: the probability of loss ($\theta = 1$),

$$L_1(F_{r_X}; r) = \int_a^r dF_{r_X}(w) \equiv F_{r_X}(r), \quad (4)$$

which is the probability that asset returns are less than the reference point r and the percentage loss ($\theta = 2$),

$$L_2(F_{r_X}; r) \equiv \int_a^r [r - w] dF_{r_X}(w), \quad (5)$$

which is the expected value of losses below the reference point r .

These indices can be used to rank loss profiles of various investment strategies to see which is riskier. The loss ordering (\mathbf{L}_θ) between F_{r_X} and F_{r_Y} for a positive integer θ and a reference point $r \in [a, b]$ can be made by comparing $L_\theta(F_{r_X}; r)$ with $L_\theta(F_{r_Y}; r)$. F_{r_X} unambiguously has less loss than F_{r_Y} relative to the loss index $L_\theta(F; r)$ with the reference point $r \in [a, b]$, denoted by

$$F_{r_X} \mathbf{L}_\theta F_{r_Y}, \quad (6)$$

if $L_\theta(F_{r_X}; r) \leq L_\theta(F_{r_Y}; r)$ for all $r \in [a, b]$ with at least one strict inequality.

The loss ordering criterion $F_{r_X} \mathbf{L}_\theta F_{r_Y}$ is consistent with stochastic dominance of degree θ . An advantage of the latter is that there is no need to specify the reference point r . Following Foster and Shorrocks (1988), define the distribution function $F_{r_X,1} \equiv F_{r_X}$ and the integrated distribution function of degree θ , $F_{r_X,\theta} \equiv \int_a^w F_{r_X,\theta-1}(t)dt$, recursively for any integer $\theta \geq 2$. Let stochastic dominance of degree θ of r_X over r_Y be $F_{r_X} \mathbf{D}_\theta F_{r_Y}$:⁶

$$F_{r_X} \mathbf{D}_\theta F_{r_Y} \tag{7}$$

if $F_{r_X,\theta}(w) \leq F_{r_Y,\theta}(w)$ for all $w \in [a, b]$ with at least one strict inequality. With repeated integration by parts, $L_\theta(F_{r_X}; r)$ can be written as⁷

$$L_\theta(F_{r_X}; r) = \int_a^r (r-w)^{\theta-1} dF_{r_X}(w) = (\theta-1)! F_{r_X,\theta}(r). \tag{8}$$

A comparison of conditions (6) and (7) via equation (8) indicates that, for any positive integer θ , r_X dominates r_Y in loss ordering $[F_{r_X} \mathbf{L}_\theta F_{r_Y}]$ if and only if r_X dominates r_Y based on the corresponding stochastic dominance ordering $[F_{r_X} \mathbf{D}_\theta F_{r_Y}]$. In other words, the loss ordering \mathbf{L}_θ is precisely the stochastic dominance ordering \mathbf{D}_θ .

The next question is to specify the value of θ . It is then natural to consider second-degree stochastic dominance ($\theta = 2$) since this case includes both the probability of loss and the percentage loss. If, for example, $F_{r_X} \mathbf{D}_2 F_{r_Y}$ with r_X being a value portfolio return and r_Y a growth portfolio return, this would certainly be consistent with myopic loss aversion for any reference point r .

2.2 A Statistical Procedure

According to the general definition of stochastic dominance of degree θ given in (7), the second-degree stochastic dominance relationship between r_X and r_Y ($F_{r_X} \mathbf{D}_2 F_{r_Y}$) can be established if

$$F_{r_X,2}(w) \leq F_{r_Y,2}(w) \quad (9)$$

for all $w \in [a, b]$ with at least one strict inequality. Let the integrated quantile function of second-degree for r_X and r_Y be $Q_{r_X,2}(p) \equiv \int_0^p Q_{r_X}(t)dt$ and $Q_{r_Y,2}(p) \equiv \int_0^p Q_{r_Y}(t)dt$, respectively, for $p \in [0, 1]$. The inequality (9) is equivalent to:

$$Q_{r_X,2}(p) \geq Q_{r_Y,2}(p) \quad (10)$$

for all $p \in [0, 1]$ with at least one strict inequality. Thus r_X dominates r_Y in the second-degree if either condition (9) or condition (10) is satisfied.⁸ In other words, if condition (9) or condition (10) is satisfied, then $F_{r_X} \mathbf{D}_2 F_{r_Y}$, indicating $F_{r_X} \mathbf{L}_2 F_{r_Y}$, and hence that the return on portfolio X , r_X , *outperforms* (or has less loss than) the return on portfolio Y , r_Y .

Condition (9) or condition (10) must be interpreted within a corresponding statistical procedure in order to draw valid inferences. Since the choice between the two conditions is primarily a matter of convenience, the statistical procedure based on integrated quantile functions [condition (10)] is selected because it is conceptually simpler.⁹

Let r_Z represent a random vector comprising the two random returns r_X and r_Y . Realizations of r_Z are assumed to be generated by a strictly stationary alpha-mixing process $\{z_t\}$ within which there are the two components $\{x_t\}$ and $\{y_t\}$. The alpha-mixing assumption is used to characterize dependence among observations which dies out as the distance between them increases. Such dependence allows observations within each of the two component sequences to be correlated, while also permitting corresponding finite samples $\{x_t\}$ and $\{y_t\}$ to be associated.¹⁰ Typically, financial series demonstrate these

two properties, which are quite distinct from independent samples of i.i.d. random variables. Such i.i.d. random variables are presumed in all existing tests for stochastic dominance [see, for example, Kaur, Rao and Singh (1994), Anderson (1996), Xu (1997), Dardanoni and Forcina (1999), Davidson and Duclos (2000), Barrett and Donald (2003), and Murasawa and Morimune (2004)], with the sole exception of Fisher, Willson, and Xu (1998). The test used in the paper represents a generalization of the last citation to alpha mixing.

The random vector r_Z can be characterized by the bivariate distribution function F_{r_Z} which is absolutely continuous in each of its two components in the neighborhood of any point in its support. F_{r_Z} is presumed to admit of a differentiable continuous density function f_{r_Z} which is positive and finite.¹¹ The quantile function corresponding to F_{r_Z} is Q_{r_Z} and the $2K \times 1$ vector of quantiles at K suitably chosen abscissae $P = \{p_i : i = 1, 2, \dots, K; 0 < p_1 < p_2 < \dots < p_K \leq 1\}$ is written

$$\begin{aligned} Q_{r_Z}(P) &= \left[Q_{r_X}(p_1), Q_{r_X}(p_2), \dots, Q_{r_X}(p_K) : Q_{r_Y}(p_1), Q_{r_Y}(p_2), \dots, Q_{r_Y}(p_K) \right]' \\ &= \left[Q_{r_X}(P)' : Q_{r_Y}(P)' \right]'. \end{aligned}$$

Let $\{x_t\}_{t=1}^T$ and $\{y_t\}_{t=1}^T$ denote observations from r_X and r_Y , respectively. The observations can be arranged in ascending order $x_{(1)} \leq x_{(2)} \leq \dots \leq x_{(T)}$ to form a $T \times 1$ vector x and $y_{(1)} \leq y_{(2)} \leq \dots \leq y_{(T)}$ to form a corresponding vector y . The sample quantiles at any point p , $0 < p \leq 1$, for r_X and r_Y are denoted $\hat{Q}_{r_X}(p) = x_{([Tp])}$ and $\hat{Q}_{r_Y}(p) = y_{([Tp])}$, $[Tp]$ being the largest integer that is less than or equal to Tp .¹²

The empirical marginal distribution functions are denoted \hat{F}_{r_X} and \hat{F}_{r_Y} , and these are used to define $m_{x_{([Tp_i])}} = \sqrt{T} [\hat{F}_{r_X}(Q_{r_X}(p_i)) - p_i]$ and $m_{y_{([Tp_i])}}$ in a corresponding way. Two $K \times 1$ vectors m_{r_X} and m_{r_Y} comprise elements $m_{x_{([Tp_i])}}$ and $m_{y_{([Tp_i])}}$ respectively. Putting these together yields

$$m_{r_Z} = \left[m'_{r_X} : m'_{r_Y} \right]'.$$

By the Yokoyama Central Limit Theorem (YCLT) for sample quantiles from an alpha-mixing sequence [Yokoyama (1978)]:

$$\lim_{T \rightarrow \infty} E(m_{r_Z}) = 0, \quad \lim_{T \rightarrow \infty} E(m_{r_Z} m'_{r_Z}) = V,$$

V being a $2K \times 2K$ positive-definite matrix of finite elements, and, as $T \rightarrow \infty$,

$$\sqrt{T} [\widehat{Q}_{r_Z}(P) - Q_{r_Z}(P)] \xrightarrow{d} N(0, \Lambda) \quad (11)$$

in which $\Lambda = D^{-1}V(D')^{-1}$ with

$$D = \text{diag} \left[f_{r_X}(Q_{r_X}(p_1)), \dots, f_{r_X}(Q_{r_X}(p_K)) ; f_{r_Y}(Q_{r_Y}(p_1)), \dots, f_{r_Y}(Q_{r_Y}(p_K)) \right].$$

The complication with D^{-1} could have been avoided by working with $\int_a^w [F_{r_Y}(t) - F_{r_X}(t)] dt \geq 0 \forall w \in [a, b]$. However this is not a serious complication here since functions of the elements of Λ are estimated directly by the moving-block bootstrap.

The integrated sample quantiles of second-degree are given by $\widehat{Q}_{r_X,2}(p_i) = \frac{1}{T} \sum_{j=1}^{r_i} x_{(j)}$ and $\widehat{Q}_{r_Y,2}(p_i) = \frac{1}{T} \sum_{j=1}^{r_i} y_{(j)}$ with $r_i = [Tp_i]$.¹³ For the K separate points of P , these are written

$$\widehat{Q}_{r_Z,2}(P) = [\widehat{Q}_{r_X,2}(P)', \widehat{Q}_{r_Y,2}(P)']'.$$

By the YCLT, integrated sample quantiles $\widehat{Q}_{r_Z,2}$ are constant weighted sums of normal variates, and hence, as $T \rightarrow \infty$, will converge to a multivariate normal distribution,¹⁴ that is

$$\sqrt{T} [\widehat{Q}_{r_Z,2}(P) - Q_{r_Z,2}(P)] \xrightarrow{d} N(0, \Omega) \quad (12)$$

where Ω is a positive-definite matrix of finite elements which are functions of the elements of Λ . This result facilitates the construction of a test-statistic for second-degree stochastic dominance.

The statistical procedure for ranking losses is developed as follows: $\widehat{Q}_{r_X,2} - \widehat{Q}_{r_Y,2}$ is the difference between the estimated integrated quantile functions of r_X and r_Y , which has dispersion $\frac{1}{T}H\Omega H'$, in which Ω is the dispersion in (12) and $H = [I_K; -I_K]$. To construct the test-statistic, a consistent estimate $\frac{1}{T}H\widehat{\Omega}H'$ is required. The moving-block bootstrap is used to provide a consistent estimate.¹⁵ If the null hypothesis that there exists a weak dominance relationship of r_X over r_Y , or $H_0: Q_{r_X,2} - Q_{r_Y,2} \geq 0$, is tested against the alternative hypothesis that there exists no weak dominance relationship between the two returns, or $H_a: Q_{r_X,2} - Q_{r_Y,2} \not\geq 0$,¹⁶ the test-statistic for second-degree stochastic dominance c is defined as:

$$c = \Delta' \left[\frac{1}{T} H \widehat{\Omega} H' \right]^{-1} \Delta, \quad (13)$$

In (13), $\Delta = [(\widehat{Q}_{r_X,2} - \widehat{Q}_{r_Y,2}) - (\widetilde{Q}_{r_X,2} - \widetilde{Q}_{r_Y,2})]$; $\widehat{Q}_{r_X,2}$, $\widehat{Q}_{r_Y,2}$, and $\widehat{\Omega}$ are unrestricted estimators while $\widetilde{Q}_{r_X,2}$ and $\widetilde{Q}_{r_Y,2}$ are the restricted estimators under H_0 . The restricted estimators are computed by minimizing

$$\begin{aligned} & [(\widehat{Q}_{r_X,2} - \widehat{Q}_{r_Y,2}) - (Q_{r_X,2} - Q_{r_Y,2})]' \left[\frac{1}{T} H \widehat{\Omega} H' \right]^{-1} [(\widehat{Q}_{r_X,2} - \widehat{Q}_{r_Y,2}) - (Q_{r_X,2} - Q_{r_Y,2})] \\ & \text{s.t. } (Q_{r_X,2} - Q_{r_Y,2}) \geq 0. \end{aligned}$$

The test-statistic c of equation (13) is asymptotically distributed as the weighted sum of χ^2 -variates, known as the $\bar{\chi}^2$ -distribution [see, for example, Robertson, Wright and Dykstra (1988) and Shapiro (1985, 1988)]. Upper and lower critical bounds for testing inequality restrictions with the test-statistic c are provided by Kodde and Palm (1986). These bounds are given by

$$\alpha_l = \frac{1}{2} Pr(\chi_1^2 \geq q_l), \quad (14)$$

and

$$\alpha_u = \frac{1}{2}Pr(\chi_{K-1}^2 \geq q_u) + \frac{1}{2}Pr(\chi_K^2 \geq q_u), \quad (15)$$

where q_l and q_u are the lower- and upper-bounds, respectively, for the critical values of the test-statistic c . A lower-bound is obtained by choosing a significance level, α , and setting degrees of freedom (df) equal to one. An upper-bound is obtained by choosing the significance level, α , and setting df equal to K . Thus, decision rules based on the statistic c depend on whether c exceeds the upper-bound or c is smaller than the lower-bound within a procedure now to be described.¹⁷

Given the test is a test for weak dominance, to draw an inference for a strict dominance relationship, we need a two-step procedure. First, $H_0^{(1)} : Q_{r_X,2} - Q_{r_Y,2} \geq 0$ is tested against $H_a^{(1)} : Q_{r_X,2} - Q_{r_Y,2} \not\geq 0$. The test-statistic for this is $c^{(1)}$. If $c^{(1)} < q_l$, then $H_0^{(1)}$ is not rejected and this provides evidence that F_{r_X} weakly dominates F_{r_Y} in the second degree. Second, $H_0^{(2)} : Q_{r_Y,2} - Q_{r_X,2} \geq 0$ is tested against $H_a^{(2)} : Q_{r_Y,2} - Q_{r_X,2} \not\geq 0$ using the test-statistic $c^{(2)}$. If $F_{r_X} \mathbf{D}_2 F_{r_Y}$, then $H_0^{(2)}$ cannot be true. Hence, if $c^{(2)} > q_u$ in addition to $c^{(1)} < q_l$, then $H_0^{(1)}$ is not rejected while $H_0^{(2)}$ is and it can be concluded that $F_{r_X} \mathbf{D}_2 F_{r_Y}$. However, if $c^{(1)} < q_l$ and $c^{(2)} < q_l$, then no strict dominance relationship can be established. The procedure for testing $F_{r_X} \mathbf{D}_2 F_{r_Y}$ is directly applicable to testing for $F_{r_Y} \mathbf{D}_2 F_{r_X}$. The decision rules for the tests are summarized in Table 1.

3 Investment-Style Portfolios and Indices

Barr Rosenburger, econometrician and former finance professor at the University of California, Berkeley, founded a firm which is now called BARRA in the 1970's. BARRA has grown into a worldwide consulting organization and since the 1970's it has maintained style portfolio indices for the United States and Canada. The monthly returns of these indices are published monthly. Each index tracks the performance of a portfolio and each portfolio is rebalanced semi-annually according to various style criteria in line with

Fama and French (1992, 1996, 1998) and Sharpe (1988, 1992). The American indices comprise large capitalization (large-cap or LC), medium capitalization (mid-cap or MC) and small capitalization (small-cap or SC) index portfolios; for Canada there are only two classes of portfolios: LC and SC. In total, therefore, there are six American style indices: a value and a growth index for each of LC, MC and SC. For Canada there are four indices: LC growth and value, SC growth and value. The American growth and value indices are available as follows: LC from January 1975, MC from June 1991 and SC from January 1994. The Canadian LC indices are available from January 1982; the SC indices from July 1990. All indices used in the empirical work end in August 2000. These financial series are suitable to answer the question—“Which is riskier, value or growth?”—in the North American context.

The American LC growth and value indices are constructed by dividing the stocks in the S&P 500 Index according to book-to-price ratios. The LC value index contains approximately fifty percent of the S&P 500 stocks with the largest book-to-price ratios, while the LC growth index consists of the other fifty percent of the S&P 500 stocks with the lowest book-to-price ratios. Rebalancing takes place on January 1 and July 1 of each year, based on book-to-price ratios and market capitalizations at the close of trading one month before (i.e., November 30 and May 31). To permit the indices to have a timely influence on the composition of a mimicking portfolio, the newly rebalanced indices become effective one month later. Similarly, the American MC (SC) growth and value indices are constructed by dividing the stocks in the S&P MidCap 400 Index (the S&P SmallCap 600 Index) into growth and value stocks based on book-to-price ratios.

The Canadian style indices are constructed similarly. BARRA divides the five hundred largest capitalization stocks in the Canadian equity market into two mutually exclusive groups: large-cap (the top two hundred) and small-cap stocks (the remaining three hundred). Each group is then divided according to an overall weighted rank which comprises two-thirds of the book-to-price rank and one-third for the dividend yield rank.¹⁸

The value index is then made up of the stocks of the highest overall rank up to a cut-off point where the index is equal to fifty percent of the float capitalization of each group.¹⁹ The growth index is made up of the remaining stocks. The stocks covered by each index are rebalanced on December 31 and June 30, based on the book-to-price ratio, dividend yield and market capitalization at the close of trading. As with the American portfolios, and for the same reason, the newly rebalanced indices become effective one month later (i.e., July 31 and January 31). In addition, the indices are also adjusted each month to reflect changes in the composition of the universe.²⁰

Investing in a BARRA value portfolio is clearly an example of widely diversified value investment with infrequent portfolio rebalancing; indeed since a BARRA value portfolio requires continual re-investment in the lower segment of the market for any given capitalization, it is arguable that downside risk has been minimized while the corresponding diversification has been made as wide as possible. Thus riskiness can be evaluated by comparing the performance of the BARRA value portfolio relative to the corresponding growth portfolio, on the basis of myopic loss aversion. Although established methods (such as Sharpe's index, Treynor's performance index or Jensen's excess returns) may be used to rank assets, these methods are generally based on the first two moments of asset-return distributions which are assumed to be independently and identically normal. As shown later in the paper, these assumptions are unrealistic for the financial data of this paper; in particular, the first two moments do not provide sufficient information on loss ranking.

Each style index measures a monthly total return, that is, the return, in terms of dividends and capital gains or losses, to the market value of each portfolio at the beginning of the period. By definition, the value portfolio comprises undervalued stocks and the growth portfolio comprises growth stocks which have relatively high prices. Of course, overvaluation and undervaluation cannot generally persist for each and every stock. Market activity will ensure that the prices of some undervalued stocks will rise and

that of some overvalued stocks will fall, relative to their true underlying value. Insofar as such movements take place, some value stocks will be reclassified as growth stocks, and some growth stocks as value stocks. Moreover, when the former constitute capital gains, these will add to the return on the value portfolio; when the latter constitute capital losses, these reduce the return on the growth portfolio.

4 Empirical Results

4.1 Background Description

The time periods covered by the growth and value indices of different markets and different capitalizations are shown in Table 2. From 1997 onwards, North American economies have experienced considerable technological change and financial markets have revealed substantial growth stock pricing bubbles. During this period, value-investment strategies seem to have been overshadowed by a euphoria for new technologies and their concomitant fast-growing growth stocks. Of course, the sample period to be used for evaluating value-investment strategies goes back further than 1997, as indicated in Table 2.

In Table 3, the American LC, MC and SC growth portfolios are revealed to have higher mean returns and higher standard deviations than the corresponding value portfolios. The returns of the American LC growth and value portfolios have large positive kurtosis, while the remaining American portfolios are smaller in this respect. If minimum return is taken as a measure of downside risk, then this appears to be smaller for each of the American value portfolios than it is for each corresponding growth portfolio.

A somewhat different picture emerges from Table 4 for the Canadian market. The Canadian LC value portfolio has a higher mean and lower standard deviation than the corresponding growth portfolio. The return on the Canadian LC growth portfolio has

larger kurtosis than its value counterpart while this position is reversed for the SC portfolios. Using minimum return, downside risk is larger for LC growth than for LC value; and larger for SC value than for SC growth.

Using the first two moments and the minimum return is, at best, an inexact measure of downside risk, especially since, in Table 5, the Jarque-Bera tests reveal that none of the return distributions is normal. In addition, returns are positively correlated between corresponding samples (Table 6) and there is some, albeit weak, evidence that observations within a sample are weakly autocorrelated (see for example, Tables 7–8). The correlations are higher between within-country indices than between cross-country series, reflecting the fact that, while returns in financial markets are generally correlated, they are less correlated than returns within an economy. Non-normality, association between samples and weak dependence within a sample of returns are accommodated by the new statistical procedures described in section 2.

It is recognized that growth and value portfolios may perform differently at different stages of a market cycle within a given market regime. From the end of 1997 onward, North American economies have experienced technological change sufficient to induce in financial markets a euphoria for exceptional growth stocks and hence growth-stock pricing bubbles and depressed value stock prices. This mirrors quite closely the flat stock performance of Berkshire Hathaway Inc., one of the best known value-investment holding companies.

4.2 Myopic Loss Ranking Results

Following the background discussion in section 4.1, two sets of myopic loss ranking results are considered: (1) the whole of each sample from the beginning of the portfolio until August 2000; and (2) a subsample of each sample from the beginning of a portfolio until December 1997. The myopic loss ranking is designed to answer the following questions:

Is a value-investment strategy riskier than the corresponding growth-investment strategy within each country? Moreover, will the answer to this question change, if the sample period includes the period December 1997–August 2000?

The differences in integrated quantiles for the American data are estimated and graphed in Figures 1–6. When the solid line (the estimated differences of integrated quantiles) is above the dotted line (the indifference line) in the figures, then the value portfolio is dominating (but not necessarily significantly dominating) the growth portfolio during the period examined. A dominance relationship appears to exist between the US LC value and growth portfolios during 1975.01–1997.12 (see Figure 2), between the US MC value and growth portfolios (see Figure 4), and between the US SC value and growth portfolios during both 1994.01–2000.08 and 1994.01–1997.12 (see Figures 5 and 6). But when the period extends to include 1998–2000, the dominance relationship disappears between the US LC value and growth portfolios during 1975.01–2000.08 and between the US MC value and growth portfolios during 1991.06–2000.08 (see Figures 1 and 3). In these cases, some, but not all, differences in estimated integrated quantiles are negative.

The differences in estimated integrated quantiles for the Canadian data can be graphed similarly. Dominance appears to exist between the Canadian LC value and growth portfolios during both 1982.01–2000.08 and 1982.01–1997.12. In these cases, all of the differences in estimated integrated quantiles are positive. The same does not hold for the Canadian SC value and growth portfolios during both 1990.07–2000.08 and 1990.07–1997.12. In these cases, some, not all, of the differences are negative.

The observed differences shown in the figures are based on the sample estimates. To evaluate the statistical significance of the differences and, therefore, to draw valid inferences, the statistical procedure described in the paper is applied. The results are given in Tables 9 and 10.²¹ At the 5% significance level, the lower- and upper-bounds of the critical value of the test-statistic are 2.706 and 17.670, respectively; at the 10% signifi-

cance level, the critical values are 1.642 and 15.337. When the value of the test-statistic is between the lower- and upper-bounds, the p -value is computed based on simulations.²² When the value of the test-statistic is less than (greater than) the lower(upper)-bound of the critical value for a chosen significance level, the weak dominance relationship under the null hypothesis cannot (can) be rejected. When the value of the test-statistic is greater than the lower-bound and less than the upper-bound, the decision is made by comparing the p -value with the chosen significance level, say 10%. If the p is greater than 10%, then a weak dominance relationship under the null hypothesis is not rejected; otherwise, the null hypothesis is rejected. A weak dominance relationship in one direction but not in the other implies a strict dominance relationship in the one.

The results in Table 9 show that the American LC value portfolio does not strictly dominate its growth counterpart during 1991.06–1997.12 because neither LC value weakly dominates LC growth nor LC growth weakly dominates LC value at the 10% significance level (the p -value is 0.2644). The test result indicates that the difference in loss risk between two styles is not statistically significant. The American MC value portfolio strictly dominates its growth counterpart during 1991.06–1997.12 because MC value weakly dominates MC growth but MC growth does not weakly dominate MC value at the 10% significance level (the p -value is 0.0625). The American SC value portfolio strictly dominates SC growth during 1994.01–1997.12 because SC value weakly dominates SC growth but the opposite is not statistically significant at the 10% level (the p -value is 0.0828). As soon as the sample period is extended to 2000.08, no strict dominance relationship exists for all cases. These results show that generally the American MC and SC value portfolios are less risky, in terms of downside risk, than the corresponding growth counterparts. But in the period in which there were growth stock pricing bubbles, value strategies were as risky as growth strategies.

The results in Table 10 show that only the Canadian LC value portfolio strictly dominates the LC growth portfolio during 1982.01–1997.12 because the Canadian LC

value weakly dominates LC growth counterpart but the opposite is not statistically significant at the 10% level (the p -value is 0.0676). However, no such strict dominance relationship exists for SC cases or for both LC and SC cases when the data of 1998.01–2002.08 are added to the sample. These results indicate that the period 1998.01–2000.08 is one in which value portfolios did not offer as large a margin of safety as growth portfolios.

In order to appreciate the different results across the United States and Canada, it is useful to compare different capitalizations in the two markets. In the United States, SC stocks generally refer the stocks with capitalizations of US \$ 250 million – US \$ 1 billion, MC stocks US \$ 1 billion – US \$ 5 billion, and LC stocks US \$ 5 billion or more. The capitalization range for American stocks is set between US \$ 250 million and US \$ 5 billion and beyond. In Canada, this range is set much lower; that is, from less than CN \$ 100 million to CN \$ 1 billion and beyond. Canadian LC stocks generally refer to stocks with capitalization of CN \$ 1 billion or more and they are considered MC stock equivalents in the United States. Any capitalizations less than CN \$ 1 billion are considered SC stocks in Canada. In the Canadian SC category, some Canadian stocks are truly American SC equivalents while others are smaller capitalization stocks. This may explain why American MC and SC value portfolios and Canadian LC, not SC, value portfolio are less risky than their growth counterparts.

Evidently the research reported here demonstrates that both value and growth strategies can be risky. For much of the past considered here, MC and SC value strategies in the United States and LC value strategy in Canada appear to offer a larger margin of safety and hence are less risky than their growth counterparts in terms of downside risk. This conclusion is robust across countries but not entirely robust over time.

5 Concluding Remarks

This paper has examined the risk of value investment in relation to the risk of growth investment by considering a myopic loss-averse investor who holds a widely diversified portfolio, relies on infrequent portfolio rebalancing and seeks a large margin of safety. This type of investor has been characterized by way of a well-defined loss profile which has been shown to engender a loss ranking; that is, a means by which alternative portfolios may be ranked according to their losses. Losses have been interpreted as returns on the low side of a cut-off point of the returns distribution. Since the cut-off point is impossible to estimate, returns have been considered below every possible cut-off point. This approach is quite new and differs markedly from conventional approaches which focus either on the first two moments of the returns distribution, or on the value risk premium in an augmented CAPM in which proper allowance has been made for broad market risk.

Within this general background, the achievements of the paper may be summarized as follows:

1. The loss profile (and hence the loss ranking) has been developed to characterize the class of myopic loss-averse (or MLA) investors and thereby the performance of different portfolios. This approach is undoubtedly novel and permits an evaluation of risk from an MLA viewpoint, without imposing further restrictions on the parametric form of the utility function or the return generating process.
2. The loss profile has been shown, for every cut-off point, to be equivalent to second-degree stochastic dominance. This result, which is ironically borrowed from poverty analysis, is nevertheless applicable to the analysis of financial data.
3. A suitable new statistical procedure has been developed which allows two different, but related, portfolios to be ranked (within a given level of significance) according to their losses. The new procedure has two specific features:

- (i) Most importantly, it is not restricted to independent samples of i.i.d. data but permits application to weakly dependent data in two associated samples. To our knowledge, no other test for second-degree stochastic dominance has yet been developed for this class of data. Such data are common in finance and so this feature represents a notable step forward.
 - (ii) The statistical procedure does not specify the null hypothesis as one of equality, since rejection leaves open whether second-degree stochastic dominance has been established. Rather, it formulates second-degree stochastic dominance as an inequality to be tested under the null hypothesis. This formulation follows the same path as Fisher, Willson, and Xu (1998) which is also used by Dardanoni and Forcina (1999) and, more recently, Murasawa and Morimune (2004).
4. The statistical procedure has been applied to data which manifestly exhibit the features summarized in 3(i) above and is used to determine whether value portfolios are more risky (according to downside risk) than corresponding growth portfolios, across economies and over time. During the period 1998.01–2000.08, value investment has been found to be no more risky than growth investment; but during the period ending 1997.12 value investment in the US, for medium and small capitalization portfolios, was less risky than corresponding growth investment. A similar result holds for the large capitalization value and growth portfolios in Canada for the same period.

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Notes

¹Lakonishok, Schleifer and Vishny (1994) and Dichev (1999) also demonstrate respectively that fundamental and bankruptcy risk cannot fully explain the higher returns on value stocks. Similarly, Anderson, Korsun and Murrell (2003) use non-market data from a privatization program to reveal that risk cannot fully explain the higher returns on value stocks.

²This statistical procedure has two special features. The first feature is that the test is not limited to independent samples of independently and identically distributed data, as in McFadden (1989), Klecan, McFadden and McFadden (1991), Kaur, Rao and Singh (1994), Anderson (1996), Xu (1997), Davidson and Duclos (2000), Barrett and Donald (2003) and Murasawa and Morimune (2004), but permits application to weakly dependent data in associated samples. This is achieved by extending Fisher, Willson and Xu (1998) to data generated by alpha-mixing and then making use of a little-known central limit theorem to find the distribution of the test statistic. The second feature is that the new test has an appropriate null hypothesis. Unlike Tolley and Pope (1988), McFadden (1989) and Anderson (1996), it does not use a null hypothesis of equality since then rejection leaves open whether stochastic dominance has been established [see Levy (1992, p. 547)]; rather the test formulates stochastic dominance as an inequality to be established [see Xu (1994), Fisher, Willson, and Xu (1998), and Dardanoni and Forcina (1999)].

³The ordering of loss profiles based on a loss index corresponds precisely to the ordering of loss profiles based on stochastic dominance. Xu and Osberg (1998) address a similar problem in a different context. Our discussion is also very much influenced by Benartzi and Thaler (1995) and Foster, Greer, and Thobcke (1984).

⁴Let $F(w) = p$, $0 \leq p \leq 1$ when $w \in [a, b]$, $-\infty < a < b < \infty$. If F is strictly monotonic, the inverse function of F is given by $w = F^{-1}(p) = Q(p)$; thus $Q(p)$ is

given by $Pr[w \leq Q(p)] = p$. If the distribution function is weakly monotonic, then $Q(p) = \inf\{w : F(w) \geq p, 0 \leq p \leq 1\}$.

⁵This is similar to the value at risk and can be referred to as the “return” at risk.

⁶Note that, for income distributions, Foster and Shorrocks (1988) require $w > 0$. Fishburn (1976) requires $w \in [0, b]$ where $b \in \mathbf{R}_+$. O’Brien (1984) requires $w \in \mathbf{R}$.

⁷This result is attributed to Riemann and Liouville [see Zygmund (1959, p. 133)].

⁸See Hadar and Russell (1969), Hanoch and Levy (1969), Rothschild and Stiglitz (1970), and Bawa (1975) for definitions based on distribution functions, and Levy and Kroll (1978) for the definitions based on quantile functions.

⁹The domain of the integrated quantile functions for both random variables being compared is the interval $[0, 1]$, while the domains of the integrated distribution functions are case-specific and are generally not the same; this increases the possibility of ‘sampling zeros’ in tests based on sample proportions.

¹⁰See, for example, Bierens (1994) and Davidson (1994).

¹¹This allows for a convenient representation for the limiting variance-covariance matrix of the sample quantiles, and could be relaxed, although at the cost of additional complexity [see Serfling (1980)].

¹²If S represents a $T \times K$ selection matrix with unity in the $[Tp_i]$ th position of column i , $i = 1, 2, \dots, K$, zeros elsewhere, then $\widehat{Q}_{r_X}(P) = [x_{([Tp_1])}, x_{([Tp_2])}, \dots, x_{([Tp_K])}]' = S'x$, and $\widehat{Q}_{r_Y}(P) = [y_{([Tp_1])}, y_{([Tp_2])}, \dots, y_{([Tp_K])}]' = S'y$, and the $2K \times 1$ vector of sample quantiles is $\widehat{Q}_{r_Z}(P) = [\widehat{Q}_{r_X}(P)', \widehat{Q}_{r_Y}(P)']' = [I_2 \otimes S'] \begin{bmatrix} x' \\ y' \end{bmatrix}'$.

¹³These estimators may be constructed by noting that

$$Q_{r_X,2}(p_i) = \int_0^{p_i} Q_{r_X}(t) dt,$$

$$\begin{aligned}
&= \int_0^{Q_{r_X}(p_i)} w dF_{r_X}(w), \\
&= F_{r_X}(Q_{r_X}(p_i)) \int_0^{Q_{r_X}(p_i)} \frac{w dF_{r_X}(w)}{F_{r_X}(Q_{r_X}(p_i))}.
\end{aligned}$$

Thus $\widehat{Q}_{r_X,2}(p_i) = \frac{p_i}{r_i} \sum_{j=1}^{r_i} x_{(j)}$. Since $[Tp_i] = r_i$, $\widehat{Q}_{r_X,2}(p_i) = \frac{1}{T} \sum_{j=1}^{r_i} x_{(j)}$. A corresponding argument applies to $\widehat{Q}_{r_Y,2}(p_i)$.

¹⁴As indicated in footnote 12, $\widehat{Q}_{r_X}(P) = S'x$ and $\widehat{Q}_{r_Y}(P) = S'y$, which together form $\widehat{Q}_{r_Z}(P)$. The matrix S here is conveniently defined: (i) to be representative of the range of x and y , and (ii) to ensure that the sample quantiles \widehat{Q}_{r_X} and \widehat{Q}_{r_Y} provide good representations of the true quantile functions Q_{r_X} and Q_{r_Y} . Nevertheless, S may be varied to form different sets of quantiles, so long as each variant admits of requirements (i) and (ii) above. Indeed the observations $z = [x':y']'$ may themselves be viewed as selected quantiles from larger corresponding samples from r_X and r_Y , and hence as having, by the YCLT, a multivariate normal distribution in a sense analogous to (11).

¹⁵See Kunsch (1989) and Liu and Singh (1992).

¹⁶Xu, (1994), Xu (1997), Fisher, Willson and Xu (1998), and Xu and Osberg (1998) proposed and used the same form of hypotheses for stochastic dominance; this is referred to as the ‘‘Wolak procedure’’ in Davidson and Duclos (2000).

¹⁷As Wolak (1991) points out for nonlinear one-sided tests, if the least favorable parameter value does not satisfy the null hypothesis with equality, the test may have incorrect asymptotic size. The practical implication of this is that the upper- and lower-bounds will be valid *slack* bounds, and can still be used to draw asymptotically valid inferences.

¹⁸Dino Mastroianni, Equity Consultant, at BARRA International Canada explains BARRA’s construction method as follows: the stock with the highest book-to-price ratio is ranked number one; the same rule applies to the dividend yield; and the weighted

average of rankings is used to divide the universe into the value and growth stocks. The method of constructing indices has several advantages: (i) the book-to-price ratio and the dividend yield are easy to measure and understand, and are useful for capturing the distinction between value and growth; and (ii) the book-to-price ratio and dividend yield are relatively stable compared with other selection criteria such as the price-earnings ratio, and hence are desirable for constructing indices.

¹⁹The cut-off point is based on the market value of capitalization, and hence varies over time.

²⁰In general, stocks in an index are not removed from that index unless they cease to exist or until they are displaced through rebalancing. According to BARRA, a buffer has been created to prevent stocks from bouncing continually between the large- and small-cap subsets. The buffer comprises stocks of basically two types. First, any small-cap stock cannot join the large-cap universe unless it crosses a threshold of 120 percent of the capitalization of the smallest stock of the large-cap universe. Second, all multiple class stocks of large-cap companies go into the buffer.

²¹These are computed using the moving-block bootstrap method of 500 iterations with block size 24. These figures represent respectively a sufficient number of iterations and a sufficiently long time-dependence structure to yield reliable estimates. Ten integrated quantiles with common abscissae are used to evaluate differences in the return distributions.

²²The simulation is based on 1,000 iterations; this is sufficiently large to be reliable.

Table 1: Decision Rules for Strict Dominance

Second test for $H_0^{(2)}$ vs $H_a^{(2)}$	First test for $H_0^{(1)}$ vs $H_a^{(1)}$	
	$c^{(1)} < q_l$	$c^{(1)} > q_u$
$c^{(2)} < q_l$	No dominance established	$F_{r_Y} \mathbf{D}_2 F_{r_X}$
$c^{(2)} > q_u$	$F_{r_X} \mathbf{D}_2 F_{r_Y}$	No dominance established

Table 2: Sample Size of the Growth and Value Indices by Markets and Capitalization

Capitalization	American Market	Canadian Market
LC	January 1975—August 2000 308 observations	January 1982—August 2000 224 observations
MC	June 1991—August 2000 111 observations	Not available
SC	January 1994—August 2000 80 observations	July 1990—August 2000 122 observations

Note: LC, MC and SC refer to large, mid and small capitalization, respectively.

Table 3: Basic Statistics of the Return Series of American Investment Style Indices

Index	Mean	Std. Dev.	Skewness	Kurtosis	Minimum	Maximum
LC Growth	1.1262	4.0448	-.4095	4.4116	-22.710	14.090
LC Value	1.0290	3.6508	-.4390	5.6146	-20.320	13.620
MC Growth	1.8019	5.6594	.0702	2.1870	-20.808	19.560
MC Value	1.3100	4.0000	-.4334	3.4454	-16.378	15.302
SC Growth	1.2201	6.1579	.1518	3.3026	-20.517	20.855
SC Value	1.1840	4.3659	-.0980	1.8890	-18.038	9.5300

Note: (1) Kurtosis refers to excess kurtosis. (2) LC, MC, and SC data cover January 1975—August 2000 (308 observations), June 1991—August 2000 (111 observations), and January 1994—August 2000(80 observations), respectively. (3) Using mean and standard deviation as the basis for comparison, the value stocks did not outperform their growth counterparts. (4) The LC, MC, and SC value returns have smaller minima than their growth counterparts.

Table 4: Basic Statistics of the Return Series of Canadian Investment Style Indices

Index	Return	Std. Dev.	Skewness	Kurtosis	Minimum	Maximum
LC Growth	1.0262	5.1697	-.6437	4.5330	-26.180	17.170
LC Value	1.1251	4.3714	-.6439	3.1233	-18.710	14.470
SC Growth	1.3789	5.8323	.2877	3.7686	-19.890	27.680
SC Value	.8010	4.0358	-.9773	7.4201	-21.380	14.990

Note: (1) Kurtosis refers to excess kurtosis. (2) LC and SC data cover January 1982—August 2000 (224 observations) and July 1990—August 2000 (122 observations), respectively. (3) Using mean and standard deviation as the basis for comparison, the large cap value stocks outperformed their growth counterparts while the small cap value stocks did not. (4) The LC value returns have a smaller minimum than LC growth.

Table 5: Jarque-Bera Asymptotic LM Test for Normality for Investment Style Index Returns

Country	LC Growth	LC Value	MC Growth	MC Value	SC Growth	SC Value
US	248.1188	398.6301	19.2636	51.8897	9.7445	61.4371
Canada	196.4325	100.8090	NA	NA	66.3260	273.0825

Note: (1) The complete data sets are used for all returns. (2) The 5 percent and 10 percent critical values are 5.9915 and 4.6052, respectively. NA (Not Available) refers the fact that no data are available for the test.

Table 6: Correlation Matrix of the Return Series of American and Canadian Investment Style Indices

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1) US LC Growth	1.00							
(2) US LC Value	.76	1.00						
(3) US MC Growth	.78	.66	1.00					
(4) US MC Value	.65	.90	.70	1.00				
(5) Can. LC Growth	.60	.52	.73	.49	1.00			
(6) Can. LC Value	.57	.75	.53	.69	.66	1.00		
(7) Can. SC Growth	.26	.27	.56	.29	.77	.42	1.00	
(8) Can. SC Value	.36	.55	.51	.56	.69	.74	.67	1.00

Note: The statistics are computed based on data from June 1991 to August 2000 (111 observations). These index returns are all positively but not perfectly correlated. The within-country correlation coefficients tend to be much higher than the cross-country correlation coefficients.

Table 7: Autocorrelation Functions of the Return Series of the American LC Growth Style Index

Lags												
1–12	.00	.00	.00	.00	.00	.01	-.01	.00	.00	.00	.00	-.01
13–24	-.02	.05	-.04	-.05	.08	-.02	-.05	.08	.01	-.02	-.06	.08
25–36	.05	.07	.02	.02	.09	-.02	.00	-.05	.05	.01	.11	-.06
37–48	.01	.00	-.08	.02	.01	.08	.02	-.01	.01	.00	-.01	.00
49–60	.04	.05	.07	.00	.08	.00	-.05	-.09	.03	-.02	.00	-.10

Note: (1) The standard error for lags 1–60 is .06. (2) The numbers in bold font are greater than or equal to one standard error. (3) The autocorrelation functions indicate some weak dependence.

Table 8: Autocorrelation Functions of the Return Series of the American LC Value Style Index

Lags												
1–12	.00	.00	.00	.00	.00	.00	.00	.00	.01	.00	-.01	-.01
13–24	.02	-.02	-.05	.04	.04	.02	-.07	-.02	-.05	.07	-.03	.11
25–36	.11	.06	.03	.05	.05	-.03	-.03	-.04	-.01	.05	.08	.00
37–48	-.03	-.02	-.08	.02	.01	.05	.01	-.02	.06	.00	.02	.03
49–60	.03	.02	.05	.00	.06	-.06	-.09	-.04	.03	-.05	.01	-.04

Note: (1) The standard error for lags 1–60 is 0.6. (2) The numbers in bold font are greater than or equal to one standard error. (3) The autocorrelation functions indicate some weak dependence.

Table 9: Dominance Tests for American Investment Style Index Returns

Sample Period	Dominance Relationship	Test	p -Value
1975.01–2000.08	LC Growth dominates LC Value	1.9501	0.4091
1975.01–2000.08	LC Value dominates LC Growth	.0000	
1975.01–1997.12	LC Growth dominates LC Value	3.1662	0.2644
1975.01–1997.12	LC Value dominates LC Growth	.0000	
1991.06–2000.08	MC Growth dominates MC Value	1.7858	0.4407
1991.06–2000.08	MC Value dominates MC Growth	.0000	
1991.06–1997.12	MC Growth dominates MC Value	6.3922	0.0625
1991.06–1997.12	MC Value dominates MC Growth	.0000	
1994.01–2000.08	SC Growth dominates SC Value	4.5951	0.1287
1994.01–2000.08	SC Value dominates SC Growth	.0000	
1994.01–1997.12	SC Growth dominates SC Value	5.3472	0.0828
1994.01–1997.12	SC Value dominates SC Growth	.0000	

Note: (1) At the 5% significance level, the lower and upper bounds of the critical value of the test statistic are 2.706 and 17.670, respectively; at the 10% significance level, these are 1.642 and 15.337 [see Kodde and Palm (1986)]. (2) When the value of the test statistic is between the lower and upper bounds, the p -value is computed based on simulations. (3) When the value of the test statistic is less than (greater than) the lower (upper) bound of the critical value for a chosen significance level, the dominance relationship under the null hypothesis cannot (can) be rejected. When the value of the test statistic is greater than the lower bound and less than the upper bound, the decision is made by comparing the p -value with the chosen significance level, say 10%. If the p is greater than 10%, the dominance relationship under the null hypothesis is not rejected; otherwise, the relation is rejected. (4) A weak dominance relationship exists in one direction but not in the other implies a strict dominance relationship. (5) The results in this table show that during 1991.06–1997.12, US MC and SC value portfolios strictly dominate their growth counterparts.

Table 10: Dominance Tests for Canadian Investment Style Index Returns

Sample Period	Dominance Relationship	Test	p -Value
1982.01–2000.08	LC Growth dominates LC Value	4.1474	0.1784
1982.01–2000.08	LC Value dominates LC Growth	.0000	
1982.01–1997.12	LC Growth dominates LC Value	6.2470	0.0676
1982.01–1997.12	LC Value dominates LC Growth	.0000	
1990.07–2000.08	SC Growth dominates SC Value	1.8171	0.4427
1990.07–2000.08	SC Value dominates SC Growth	.3260	
1990.07–1997.12	SC Growth dominates SC Value	1.7904	0.4497
1990.07–1997.12	SC Value dominates SC Growth	.0031	

Note: (1) At the 5% significance level, the lower and upper bounds of the critical value of the test statistic are 2.706 and 17.670, respectively; at the 10% significance level, these are 1.642 and 15.337 [see Kodde and Palm (1986)]. (2) When the value of the test statistic is between the lower and upper bounds, the p -value is computed based on simulations. (3) When the value of the test statistic is less than (greater than) the lower (upper) bound of the critical value for a chosen significance level, the dominance relationship under the null hypothesis cannot (can) be rejected. When the value of the test statistic is greater than the lower bound and less than the upper bound, the decision is made by comparing the p -value with the chosen significance level, say 10%. If the p is greater than 10%, the dominance relationship under the null hypothesis is not rejected; otherwise, the relation is rejected. (4) A weak dominance relationship exists in one direction but not in the other implies a strict dominance relationship. (5) The results in this table show that during 1982.01–1997.12, the Canadian LC value portfolio strictly dominates its growth counterpart.

Figure 1: Difference in Empirical Cumulative Quantiles of the American LC Value and Growth Index Returns: 1975.01–2000.08

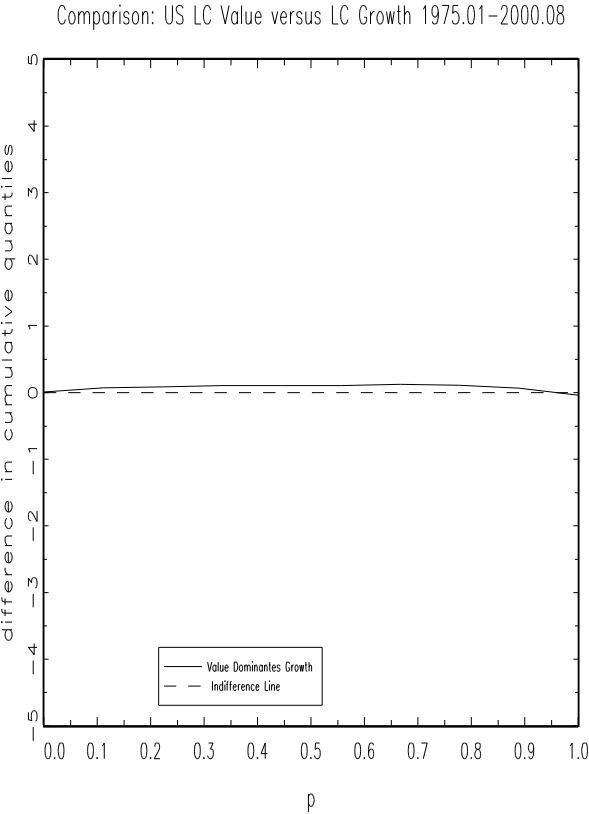


Figure 2: Difference in Empirical Cumulative Quantiles of the American LC Value and Growth Index Returns: 1975.01–1997.12

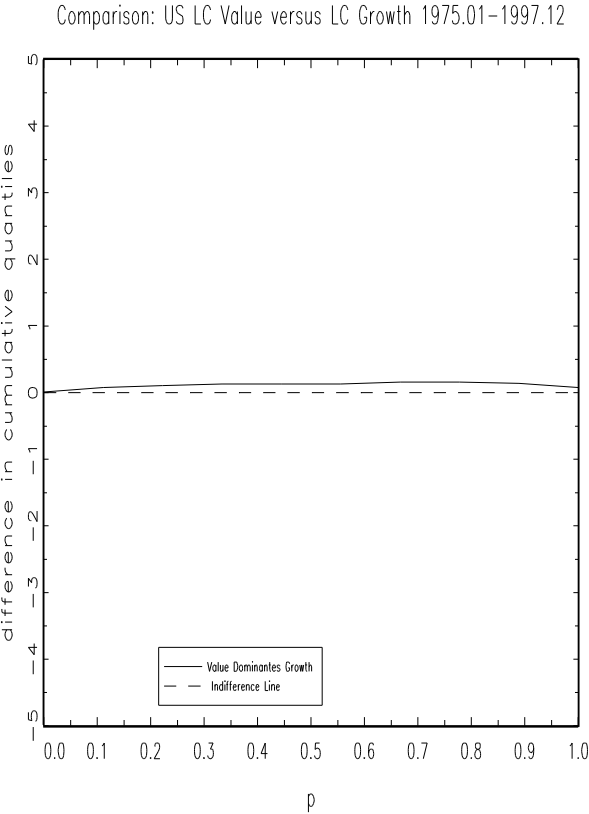


Figure 3: Difference in Empirical Cumulative Quantiles of the American MC Value and Growth Index Returns: 1991.06–2000.08

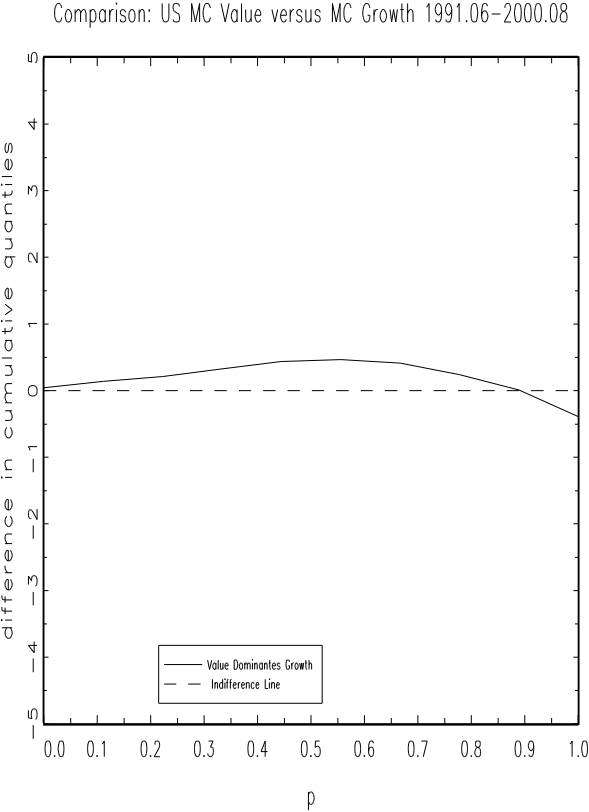


Figure 4: Difference in Empirical Cumulative Quantiles of the American MC Value and Growth Index Returns: 1991.06–1997.12

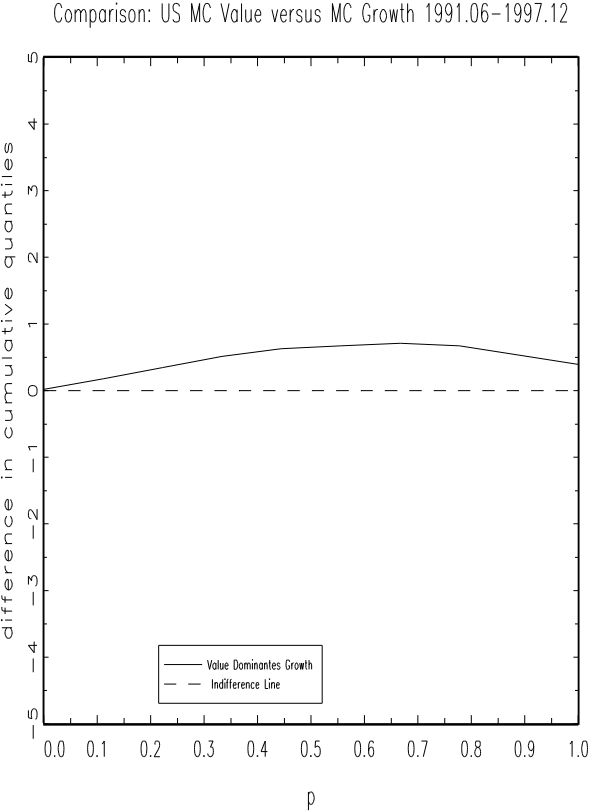


Figure 5: Difference in Empirical Cumulative Quantiles of the American SC Value and Growth Index Returns: 1994.01–2000.08

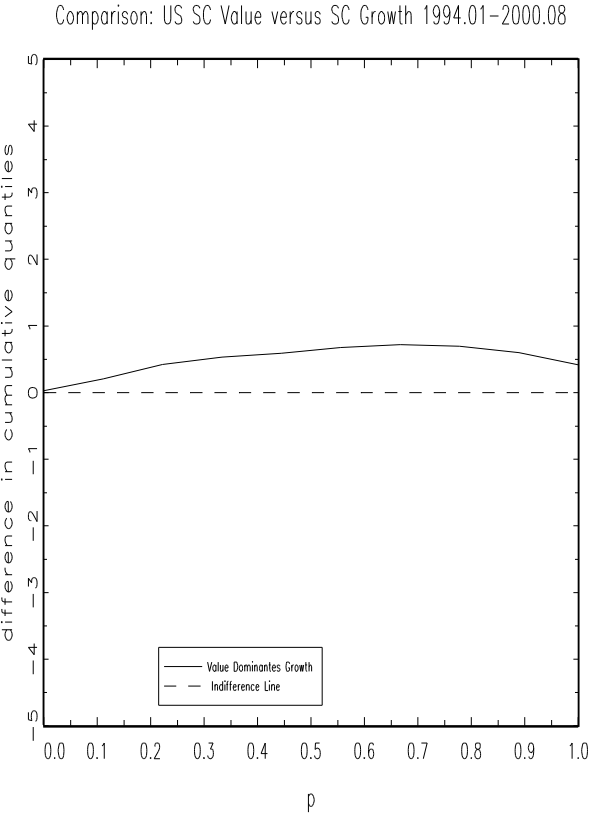


Figure 6: Difference in Empirical Cumulative Quantiles of the American SC Value and Growth Index Returns: 1994.01–1997.12

