

Chapter 9

Risk and Insurance in International Finance

While macroeconomic aspects of international finance are critical in understanding the movements in the exchange rates in the medium and long run, there are many short-term dimensions of exchange rate fluctuations that can best be understood through an understanding of microeconomic aspects of international financial markets. For instance, there are many financial instruments that are traded in financial markets, and the volume of trade in these markets and the prices that emerge in these markets have ramifications for exchange rates. It is therefore difficult to get a detailed picture of international finance without a sufficient grasp of these markets and the instruments that are traded in these markets. This chapter provides an introduction to some of these instruments: futures, options and swaps.¹

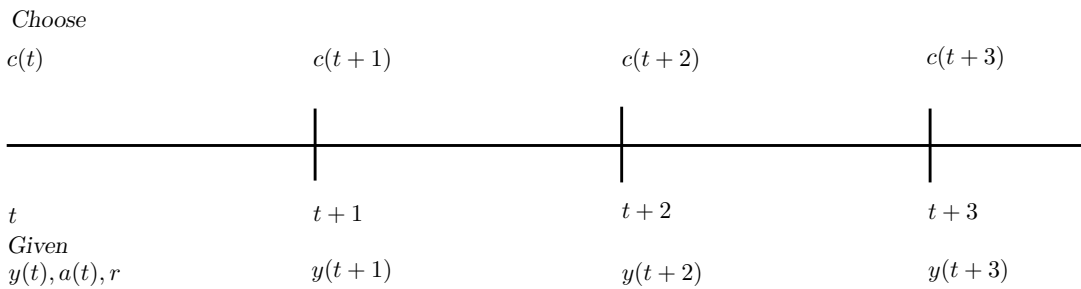
My ultimate objective in discussing these financial instruments, however, is to provide links between *uncertainty*, *risk*, and *risk management*. In principle, financial markets pool and share risks across individuals. As such, there are significant parallels between financial markets and macroeconomic policy as complementary venues for insuring against individual risks. The objective here is to introduce risk-sharing, risk-pooling, and insurance in the context of both markets and macroeconomic policy.

We discussed in Chapter 8 that one of the ultimate policy objectives is to reap the benefits of international risk sharing, and the choice about the degree of international capital mobility is a policy instrument, which serves that objective. In this chapter, I amplify the risk-sharing properties of international capital flows. International financial markets allow risks to be shared globally and over time through varied financial instruments, and it is ultimate motives for risk sharing that matter for a macroeconomic perspective. The motive for risk-sharing over time allows us to study *net* financial assets and flows across borders, and the motive for risk-sharing across states of nature allows us to study *gross* financial flows across borders, which are an order of magnitude larger than net flows (Chapter ??, Figure 1.5).

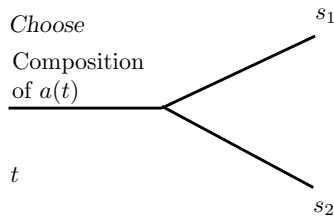
Figure 9.1 shows alternative scenarios for decision making. Figure 9.1a considers a case where an individual makes consumption–saving decisions with perfect foresight. These decisions involve choosing a (feasible) consumption sequence $\{c(t), c(t + 1), \dots\}$ given income sequence $\{y(t), y(t + 1), \dots\}$, initial value of assets $a(t)$, and a constant real interest rate r . This is the type of problem addressed by the permanent

¹This is a brief and cookbook style introduction. For more comprehensive coverage see Hull (1995), and Solnik (2000).

A) Intertemporal decision making (no uncertainty)



B) Decision making under uncertainty (no intertemporal dimension)



C) Intertemporal decision making under uncertainty

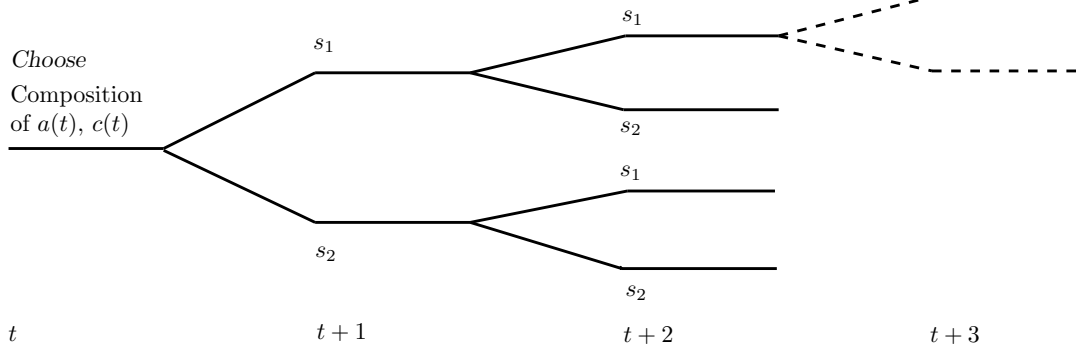


Figure 9.1: Decision making under alternative scenarios

Note: (A) There is no uncertainty (perfect foresight), and the individual chooses a consumption stream c , based on an income stream y , initial assets a , and the real interest rate r . (B) There is uncertainty, which will resolve immediately (no delay). The individual chooses the asset composition of wealth (portfolio allocation problem). (C) There is uncertainty and intertemporal dimension. The individual chooses contingent consumption and asset allocations.

income hypothesis (Blanchard and Johnson, 2010, chp. 20). We will discuss it in more detail in Section 9.2.

Figure 9.1b, on the other hand, considers a “static” decision problem under uncertainty. In this case, the problem consists of allocating wealth into different assets, say an asset with risky return (“equity”) and an asset with risk-free return (“bond”). This is a standard portfolio allocation problem, and we will study such a problem in Section 9.1.1. Finally, Figure 9.1c shows a situation in which there is recurrent uncertainty, so we have intertemporal decision making under uncertainty. In this case, the individual allocates resources over time, and across different assets. We discuss such a problem in Section 9.3.

9.1 Uncertainty and Risk

Futures contracts, options and swaps are instruments that help individuals cope with *uncertainty*, as long as trading is primarily for hedging purposes. (In speculative contexts, traders may have different beliefs about future prices. Such heterogenous beliefs are beyond the scope of these lectures.) Viewed from the perspective of underlying uncertainty, financial contracts involve trading of *risks*. In what follows, I will informally describe what I mean by uncertainty and risk, and then use these concepts to position financial contracts as risk-sharing devices.

For our purposes, it is sufficient to define uncertainty as possibility of multiple non-trivial states of nature at a future date (say “rain” or “shine”). Uncertainty may be resolved immediately, or in distant future. Each state can materialize with a non-zero probability, and probabilities summed over states at a particular date must add to one. I will assume that these probabilities are “objective,” in the sense that all individuals have the same beliefs and these beliefs coincide with the true probabilities of each state of nature in all future dates. (More realistic but technically more demanding treatments include “subjective” probabilities, and learning.) Individuals have no control over the probabilities over states of nature.

Each state of nature corresponds to a distinct outcome. While individuals cannot influence the probabilities associated with states of nature and the corresponding outcomes, they can influence the payoffs associated with each state. For instance, carrying an umbrella matters for the payoffs on sunny and rainy days, although it does not change the likelihood of rain and it does not change the intensity of rain. Since current actions determine state-dependent payoffs, they can be viewed as responses to uncertainty. The fundamental question of choice under uncertainty concerns the optimal response to uncertainty or the optimal choice of a probability distribution over state-contingent payoffs.

Individuals cope with uncertainty in a variety of ways. Self-insurance is a mechanism that is commonly used: leaving home with an umbrella is a prime example. Other mechanisms have a more collective nature: There is an “umbrella rack” in the Killam library (right across the street from where I work), and unprepared patrons can help themselves on a first-come-first serve basis. Availability is unpredictable, so there is incomplete insurance. Wheat farmers can sell wheat futures to insure themselves against fluctuations in wheat prices. Before a harvest wheat farmers have long position in wheat, and typically have a good sense of the yield. By selling wheat futures, farmers match this long position with a short position, effectively locking themselves into the futures price at the time they buy the contracts. The hedge ratio determines whether the insurance is complete.

The broader lesson here is the reallocation of risks in financial markets through financial instruments. But, above we have considered very specific cases of uncertainty, especially uncertainty originating from a single source. Also, we have introduced “risk” without being precise about it. In what follows, I will briefly discuss how economists have conceptualized risk in different contexts. The discussion will also consider risk from an individual perspective, and then broaden its scope and take a macroeconomic approach to risk and risk sharing.

9.1.1 Coping with Uncertainty at the Individual Level

Different people impute different meanings to risk. For instance, consider two financial assets with the following loosely defined characteristics

Asset 1: low expected return and low return volatility

Asset 2: high expected return and high return volatility

When asked about the relative riskiness of these assets, most individuals rank Asset 2 as the “riskier” asset. Here the ranking corresponds to ranking of volatilities. As such, risk corresponds to a single measure: standard deviation of returns (volatility). It is also independent of individual wealth, characteristics, and attitudes toward risk.

Thus, at a second glance, such a measure of risk may be unsatisfactory. In fact, consider the ranking of following return–volatility ratios

$$\text{Asset 1 Sharpe ratio} \equiv \frac{\text{low expected return}}{\text{low return volatility}} < \frac{\text{high expected return}}{\text{high return volatility}} \equiv \text{Asset 2 Sharpe ratio}.$$

Here, we have scaled average expected returns by return volatility (standard deviation). How would this information affect our ranking of Asset 1 and Asset 2 in terms of riskiness? Needless to say, we can devise alternative ways of scaling returns and combining this information with the standard deviations of returns, and our rankings may change depending on such transformations. But, the question remains: what would be a satisfactory measure of risk? To make progress and fix ideas, it is useful to start with a definition of risk.

Definition 9.1 *Risk measures the impact of a probability of a loss on well-being, either in utility or in monetary terms.*

This definition underscores that risk is a monetary or utility cost associated with uncertainty. It also suggests that risk is likely to be a “model-based” measure, in the sense that risk depends on individual characteristics and preferences. In fact, all operational measures of risk impose, either directly or indirectly, a structure on preferences.

At this point, continuing with our above example may help to reinforce ideas. Suppose, an individual investor has the following objective: maximize expected returns and minimize variance of wealth (“mean–variance investor”). How would this individual form an investment portfolio? First, although Asset 1 has the lowest variance, it would not typically be the only asset in the portfolio, because expected returns also determine total utility. Second, although Asset 2 has the highest expected return, it would also not be the only asset in the portfolio, because such a portfolio would give the highest variance of wealth. Therefore, the individual would consider a mixture of these two assets. How can we determine the proportion of each of these assets in the optimal portfolio? The answer depends on the *covariance* structure of the asset returns, or how expected returns are correlated.

A more concrete example is in order. Consider a Canadian financial investor who faces two asset-specific returns and variances: a broad Canadian equity index (TSE 300 index), and a broad global equity index (MSCI World Index). Figure 9.2 shows the mean–variance frontier from 1973 to 1998 for a fixed initial wealth portfolio made up of different combinations of these two assets. Investing in TSE index alone has relatively high variance and relatively low return. Investing in MSCI alone has relatively low variance and relatively high return. So, a comparison of these two indexes suggests that MSCI “dominated” TSE over this period. However, our model of preferences delivers a mean–variance efficient portfolio made up of 24% TSE and 76% MSCI. Thus, for a mean–variance investor a 100% MSCI is “too risky.” Individually ranking

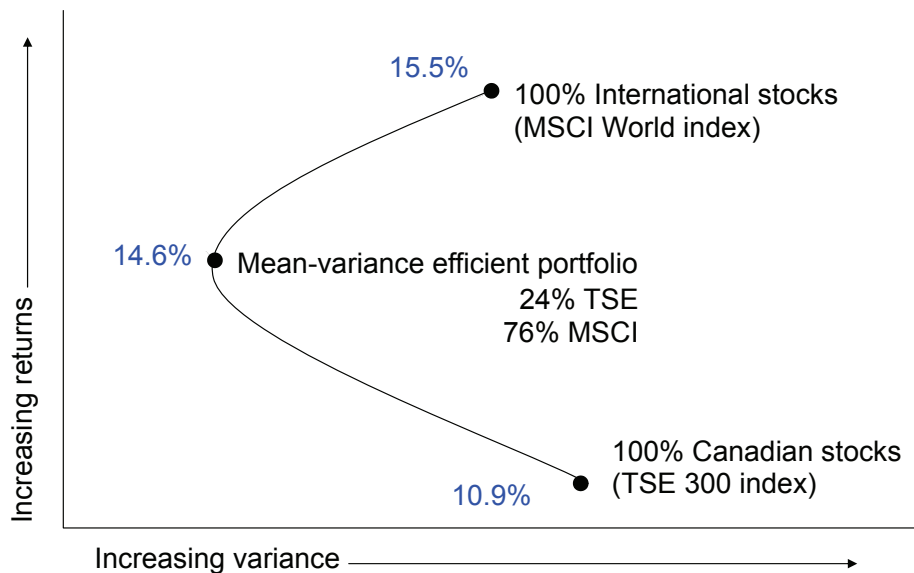


Figure 9.2: Mean–variance frontier for Canada, 1973–1998

Source: Powers (1999), based on Global Strategy Financial Inc. data.

assets from more to less risky has little practical use in this exercise—it is the variance of the entire portfolio that matters, and this variance is distinct from the variance of individual assets.

Why does covariance matter for the mean–variance efficient portfolio? It matters because TSE and MSCI are not completely synchronized, and as such volatility of aggregate returns in the mean–variance efficient portfolio is less than the volatility of individual stock indexes. In fact, covariance turns out to be a particular measure of asset-specific risk in the context of pricing individual assets, and the mean–variance investor is concerned about individual assets only to the extent that the volatility of an individual asset affects the volatility of wealth (the portfolio).

9.1.2 Demand for Risky Assets

Our analysis has demonstrated that there is demand for financial assets because they provide insurance against future contingencies, and fluctuations in future wealth. However, the above analysis does not inform us about how these returns might have been determined in the first place. It also does not inform us about whether the mean–variance efficient portfolio might change based on individual attitudes toward risk. In this section, I will address these two related questions by considering the demand side of assets (or securities) more carefully. In general, the demand for financial assets is ultimately reflected in the prices of these assets. Assets that command a high demand receive a high price and low return, and assets that command a low demand receive a low price and high return. So, understanding the demand for financial assets will help us understand asset prices. The capital asset pricing model that I will discuss next establishes such a link.

Capital Asset Pricing Model

To fix ideas, I will assume that each asset has two attributes: expected return r_i , and the covariance of this return with other assets. I will call this group of assets a “portfolio” p . The portfolio has an expected rate of return $\mathbb{E}(r_p)$ and a variance of expected returns $\text{var}(p)$. I will denote the covariance between a generic asset i and the portfolio by $\text{cov}(i, p)$. In what follows, I will develop the arguments in terms of returns on assets, but the implications of the corresponding returns for prices is immediate. I will also measure uncertain returns after normalizing them by the risk-free interest rate r , like yield on a Treasury bill.

In this contexts, two related questions arise. What determines the riskiness of a particular asset (or security)? And, how do markets price this risk? Recall that the primary objective of financial investors is to reduce fluctuations in wealth. We have also seen that a portfolio which is made up distinct assets has a smaller variance than individual assets included in the portfolio. Then, asset-specific variances are of little consequence. Rather it is the covariance of individual returns with the portfolio that matter. An asset whose return is highly correlated with the broad portfolio helps little to smooth fluctuations in wealth. By contrast, an asset whose returns correlate little with the portfolio helps smooth fluctuations in wealth. Given the preference for low wealth volatility then, those assets whose returns are highly correlated with the portfolio and thus contribute to fluctuations in wealth would face relatively low demand, and thus would command higher returns (lower prices). Similarly, those assets that help smooth fluctuations in wealth would face high demand, and thus command lower returns.

These relations are formally stated in the next expression

$$\mathbb{E}(r_i - r) = \beta \mathbb{E}(r_p - r) \quad \text{where} \quad \beta = \frac{\text{cov}(i, p)}{\text{var}(p)}. \quad (9.1.1)$$

In the above expression \mathbb{E} is the mathematical expectation conditional upon available information. An asset with a low β tends to have low covariance with the portfolio and commands low return relative to the portfolio. An asset with a high β tends to have a high covariance with the portfolio and commands a high return relative to the portfolio. Put differently, a low β asset has low returns, because it provides insurance against larger fluctuations in wealth. A high beta asset, by contrast, has to compensate for high covariance by higher return. So, in financial markets asset prices should in principle reflect the relative insurance properties of each of the assets.

Equation (9.1.1) is the celebrated *security market line* (SML) equation. In this version of CAPM investors are assumed to hold a diversified portfolio. The risk of a market portfolio is defined as its variance, and the risk of an individual security is measured by its contribution to (or its covariance with) the market portfolio. The covariances are scaled by the variance of the market portfolio and are called *betas*. The CAPM predicts that assets with higher covariances (higher risk) will command higher expected returns in equilibrium. In such a portfolio, all the idiosyncratic risk, as measured by the variance of the individual assets, would be diversified away. As such, those factors that influence the variance of the stock will not directly affect asset prices.

There are several general conclusions that emerge from the above analysis. First, risk is not independent of preferences. In the above context, preferences were structured in such a way that individuals prefer low volatility of wealth to high volatility of wealth. Second, pooling eliminates asset-specific “risk” (variance), and leaves us with an aggregate “risk”—here the variance of the returns to p . There is no way to insure

against this risk, but with sufficient diversification we can reduce asset-specific or idiosyncratic risk. Consequently, the portfolio is left with systemic (aggregate, or market) risk which cannot be diversified away. Therefore, the risky portfolio itself has to command a risk premium, given by $\mathbb{E}(r_p - r)$. In the literature, there is a debate about the appropriate size of this premium for equities.

This reasoning also leads to the two-fund separation theorem, which states that individuals with different attitude toward risk differ solely by the weights assigned to risk-free asset and the portfolio in their total wealth, but not by the composition of the portfolio. Since individuals care about the variance of their wealth, they first determine the allocation of wealth between the risky portfolio and the risk-free asset. Once this decision is made, they decide on the composition of the portfolio. As a result, all risk-averse investors in this model hold the same diversified portfolio—they only differ in terms of their allocation of wealth between risky and risk-free assets.

Derivation of Security Market Line (Optional)

To demonstrate the economic reasoning underlying the CAPM, I will provide a heuristic derivation of equation (9.1.1). The derivation of equation (9.1.1) involves the following optimality condition. Suppose the investor, who already holds a portfolio of assets, has a “small” amount of additional wealth to invest. If market prices of each of the assets in the portfolio reflect their risk adjusted returns, then the investor should be indifferent between purchasing a certain asset and purchasing *any* combination of these stocks.

In particular, assume that an investment can be financed at a risk-free interest rate r . The benefit of adding a δ amount of generic asset (i) to a portfolio is the additional expected return it brings. Suppose the change in the portfolio is represented by Δ , which changes the expected return on the portfolio r_p by the risk premium of the asset; i.e., by the difference between the expected return on the asset r_i and the cost of the financing, r . (In what follows, all returns should be interpreted in terms of expected returns, rather than actual returns.)

$$\Delta r_p = (r_i - r) \delta.$$

The marginal cost of this addition to portfolio is the change in the variance of the portfolio. To compute this change in the variance of the portfolio, let $\text{var}(p)$ denote the variance of returns on the current portfolio, let $\text{var}(i)$ stand for the variance of asset i 's returns, and let $\text{cov}(i, p)$ denote the covariance between the return of asset i and that of the portfolio p .

The return and the variance of the portfolio after adding δ of asset i are, respectively,

$$\begin{aligned} r_p + \Delta r_p &= r_p + (r_i - r) \delta, \\ \text{var}(p) + \Delta \text{var}(p) &= \text{var}(p) + 2\delta \text{cov}(i, p) + \delta^2 \text{var}(i). \end{aligned}$$

Therefore, the change in variance is

$$\Delta \text{var}(p) = 2\delta \text{cov}(i, p) + \delta^2 \text{var}(i),$$

and for δ which is sufficiently small (marginal), this approximates,

$$\Delta \text{var}(p) \simeq 2\delta \text{cov}(i, p).$$

Marginal rate of transformation (MRT) between return and variance is thus given by

$$\begin{aligned} \text{MRT}_i &= \frac{\Delta r_p}{\Delta \text{var}(p)} \\ &= \frac{(r_i - r) \delta}{2\delta \text{cov}(i, p)} \\ &= \frac{r_i - r}{2\text{cov}(i, p)}. \end{aligned}$$

Equilibrium will prevail when the opportunities provided by the market are equal to the personal valuation of this variance-return trade-off. The alternative strategy therefore involves replicating the existing portfolio by an amount δ . Such a move produces a return normalized by variance exactly analogous to the one examined above, with the difference that now the additional wealth is allocated across different assets exactly the same way as the existing portfolio: So, we have

$$r_p + \Delta r_p = r_p + (r_p - r) \delta,$$

which gives

$$\text{var}(p) + \Delta \text{var}(p) = \text{var}(p) + 2\delta \text{cov}(p, p) + \delta^2 \text{var}(p).$$

Using a similar approximation as above

$$\Delta \text{var}(p) \simeq 2\delta \text{var}(p).$$

we have

$$\text{MRT}_p = \frac{r_p - r}{2\text{var}(p)}$$

In equilibrium we would have $\text{MRT}_i = \text{MRT}_p$, which gives the desired result in equation (9.1.1).

International Capital Asset Pricing Model

The extension of the CAPM to international setting requires taking exchange rate risk into account. If PPP holds continuously (i.e., there are no capital gains and losses due to price movements) then PPP covers the exchange rate risk, and the CAPM goes through without modification. The only difference would be replacing r_p with the world portfolio r_w , which is a larger universe.

However, if PPP does not hold continuously, then the international capital asset pricing model (ICAPM) has to account for exchange rate risk which is defined with respect to the home currency as the base currency:

$$\mathbb{E}(r_i - r) = \beta_{iw} \mathbb{E}((r_w - r) + \gamma_{i1} \mathbb{E}(r_1 - r) + \dots + \gamma_{iK} \mathbb{E}(r_K - r)),$$

where $(r_i - r)$ is the risk premium on assets denominated on currencies $i = 1, \dots, K$, and γ_{ik} is the sensitivity of asset i 's return to the currency k . Risk premium is the covariance of the spot exchange rate movements with the price movements in the world portfolio which includes holdings of other currencies, and currency contracts, such as short positions on the currency futures and forwards as hedges.

Definition 9.2 *The currency hedge ratio s_{hr} is the domestic currency value of foreign securities divided by the forward currency contracts.*

If $s_{hr} < 1$, then not all the foreign exchange risk is hedged in the futures markets. What is the “best” hedge? The answer depends on the preferences of the investor. In any event, there are several issues of importance for hedging considerations. First, stock market correlations across major markets are almost identical for hedged and unhedged returns, largely because the impact of exchange rate fluctuations on domestic stock market returns is low. Second, from the viewpoint of a domestic investor, the returns in domestic currency terms on *foreign bonds* tend to be highly correlated, and currency hedge matters for bond markets. These correlations driven by the relation between the interest rates and exchange rates (the uncovered interest rate parity condition), and the fact that, in the short-run, currency risk tends to be larger than the risk (in local currency) of the corresponding bond market.

9.1.3 Risk Pooling and Risk Sharing

Capital asset pricing model is useful in clarifying several aspects of coping strategies under uncertainty and demand for risky assets at the individual level. However, the model falls short of providing a comprehensive picture of risk at the *aggregate* level, because it is not informative about the *supply* side of risky assets. From a macroeconomic perspective this is unsatisfactory for two reasons. First, prices are impossible to determine without supply and demand considerations. Second, the singular focus on the demand side of the market for the risky assets mystifies the fact that there *must* be those who supply those risky assets. The fact that there are people who supply those risky assets also imply that risks must be asymmetric across individuals, or, put differently, once pooled across individuals they roughly cancel each other.

There is another dimension to thinking about risk in a macroeconomic context: risk pooling and risk sharing are ultimately *social* mechanisms regardless of whether they are intermediated through markets or other forms of organizations such as one’s family, community, or the government. By focusing on either the supply or demand side of risky assets, it is easy to loose sight of this important social aspect of risk pooling and risk sharing, and without that insight, it would be impossible to start thinking about the risk-sharing institutions that exist in contemporary societies.

In what follows, I will describe one such mechanism: *complete financial markets*. By financial market completeness, I refer to a market in which individuals can insure against all future contingencies. Technically, I will refer to all possible future contingencies as “states of nature,” and assign an *objective* probability to all possible states with the requirement that probabilities summed at a particular date add up to one. So, there will be events classified by a pair: state and date. I will assume that individuals do not attribute any intrinsic value to states of nature above and beyond the payoffs (income) that these states correspond to.

Two aspects of the economic environment I discuss worth further commentary: market completeness, and objective probabilities. First, financial markets are *incomplete*. So, the mechanism I discuss below should not be taken as a “description” of existing financial markets. Nevertheless, exploring the implications of complete financial markets is a good starting point to think about the potential gains from reform and improvements in the existing insurance mechanisms. Second, objective probabilities over states of nature, on which all individuals can agree, are difficult to identify. States of nature often trigger behavioral responses (like anger, fear), and individuals rarely have firm basis (knowledge, information, historical probabilities) to base their opinions on future eventualities. So, it may be empirically more profitable to think about *subjective* probabilities, and how they may evolve (through learning, and teaching) over time. Nevertheless, the

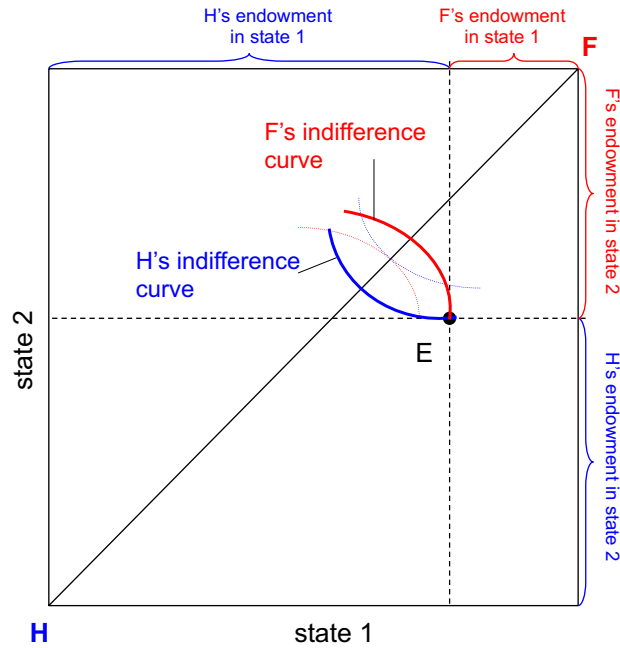


Figure 9.3: Two-state two-country endowment world

assumption of objective probabilities simplifies the analysis, and helps us to be clear about the implications our remaining premises.

With these considerations in mind, I will consider a *static* decision problem under uncertainty, which will be resolved “shortly.” This allows me to strip the problem off its intertemporal dimensions and focus on the implications of market completeness for risk sharing. To fix ideas, I will consider an environment in which there are two possible states of nature, state 1 and state 2, each corresponding to a unique level of income. In isolation, incomes in each of these states would dictate the corresponding consumption levels of that country: when income is high consumption would be high, and when income is low consumption would be relatively low. Each of these states has a probability given by

$$\pi[1] + \pi[2] = 1, \quad \text{with } 0 < \pi[s] < 1, s = 1, 2.$$

For instance, consider country H in Figure 9.3. Its endowment income pair, say $(W[1], W[2])$, in states 1 (high) and 2 (low) is point E, which is also its consumption pair. In the absence of trade with another country, country H cannot shift resources from state 1 to state 2, and vice versa. So, at best country H would reach an indifference curve that goes through point E.

Now introduce country F, also shown in Figure 9.3. Country F’s endowment pair is $(W^*[1], W^*[2])$, whereby country F has relatively high income in state 2 and low income in state 1. Country F complements H in the sense that H has comparative advantage in state 1 and country F has comparative advantage in state 2. This comparative advantage is based on the fact that country H has relatively higher income in state 1 and F has relatively higher income in state 2. Just like any other context of comparative advantage, this opens up the possibility of mutually beneficial trade. In fact, country H may have higher income and thus absolute advantage in both states, but this does not prevent these countries from engaging in mutually beneficial trade. Specifically, each allocation within the “lens” outlined by H’s and F’s indifference curves

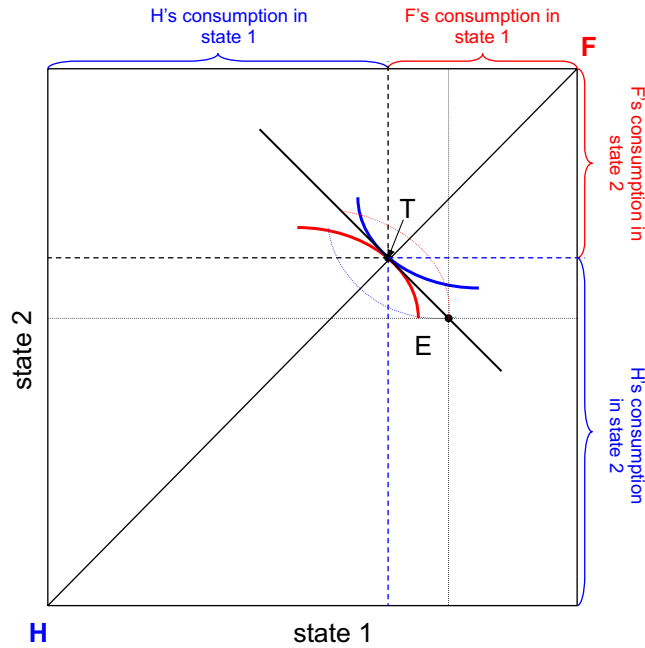


Figure 9.4: Optimal consumption allocations

Pareto dominate the allocation E.

One aspect of the square “box” in Figure 9.3 is the lack of aggregate uncertainty. Each country faces uncertainty about its income. However, regardless of the state, aggregate endowment in the global economy is constant:

$$W[1] + W^*[1] = W = W[2] + W^*[2].$$

This is the special context within which I will develop mutually beneficial trades below, but will later on briefly discuss the expected outcomes when there is aggregate uncertainty.

There are two related questions that are relevant given our focus on consumption–saving decisions. First, what is the efficient allocation of consumption across countries? Second, what is the relative price of consumption in each state? To answer the first questions we have to find a way to allocate resources across countries in each state. A complete financial market achieves this by way of trading state contingent claims. Each claim gives the buyer the right to receive one unit of the consumption good in one of the two states, and the seller the obligation to deliver one unit of the consumption good (generalizing this to more than two states is immediate). These claims can then be traded before the state is observed. Like any financial asset, these claims would have prices, emerging after a tatonnement process.

Intuitively, since country H has comparative advantage in state 1, it would be willing to sell contract which would deliver one unit of consumption good in state 1. Similarly, since country F has comparative advantage in state 2, it would be willing to sell a contract which would deliver one unit of consumption good in state 2. Since both countries would prefer to smooth their marginal utility of consumption across states, there would be demand for such contracts. Country H would buy from country F contracts that would deliver the consumption good in state 2, and country F would buy contracts from H that would deliver the consumption good in state 1. After the resolution of uncertainty—and assuming that the contracts are

honored (or enforceable)—there would be unilateral payments, leading to an *ex-ante* optimal consumption allocation. Of course, countries must agree upon such an arrangement *before* the states are actually observed. Once the uncertainty is resolved, there would be no incentive for trade in a static world.

These contracts are *state contingent claims*. One claim would simply stipulate that the seller of the claim would deliver one unit of income to the holder if state 1 materializes. Another claim would stipulate the same for state 2. This financial market is *complete*, in the sense that the number of distinct contingent claims is equal to the number of states.

Figure 9.4 illustrates the ramifications of such a financial exchange for consumption allocations (see point T). In this example both countries smooth consumption across states, whereby

$$C[1] = C[2] \quad \text{and} \quad C^*[1] = C^*[2].$$

These consumption allocations Pareto dominate the allocations endowment allocations (point E on Figure 9.4). To characterize the precise location of the allocation, we have to determine the relative prices of the state contingent claims.

Let the price the claim which delivers one unit of consumption good in state 1 by $p[1]$, and the claim which delivers in state 2 by $p[2]$. To characterize these prices, I start with the optimality condition for country H:

$$\frac{\pi[1]u'(C[1])}{\pi[2]u'(C[2])} = \frac{p[1]}{p[2]}. \quad (9.1.2)$$

This condition equates the marginal rate of substitution between the first and second state adjusted for their likelihoods of occurring (the left-hand side) to their relative price (the right-hand side). This condition treats consumption in each state as distinct consumption goods, and represents an optimal allocation of resources given the relative price.

A similar optimality condition holds for country F:

$$\frac{\pi[1]u'(C^*[1])}{\pi[2]u'(C^*[2])} = \frac{p[1]}{p[2]}. \quad (9.1.3)$$

The optimality conditions (9.1.2) and (9.1.3) have two key implications, and I discuss them in turn.

First, putting these optimality conditions together imply that

$$\frac{u'(C[1])}{u'(C[2])} = \frac{u'(C^*[1])}{u'(C^*[2])}. \quad (9.1.4)$$

Thus, as long as H and F have identical instantaneous utility functions—here meaning identical attitudes toward risk, then the ratio of their consumption across states would be identical. Moreover, given that there is no aggregate uncertainty, we have perfect consumption smoothing across states: $C[1] = C[2]$ and $C^*[1] = C^*[2]$.

The second implication of the optimality conditions, together with the assumption of no aggregate uncertainty is that prices of the contingent claims are “actuarially fair”:

$$\frac{\pi[1]}{\pi[2]} = \frac{p[1]}{p[2]}.$$

These prices support consumption allocations, which are, for each country, constant across states. Thus, countries insure against all future contingencies. We have developed consumption smoothing across states in

a two-country world with no aggregate uncertainty. A corollary to this result in the context of a small open economy is the following: when a small open economy faces actuarially fair prices of contingent claims in world markets, it would fully insure against fluctuations in consumption across states. As such, international financial markets can help pool and share risks.

Using the optimality conditions (9.1.2) and (9.1.3), we can also infer the implications of aggregate uncertainty for consumption allocations and relative prices. With aggregate uncertainty, consumption ratios across two distinct states would be constant but different from one. As such, relative prices would deviate from pure actuarially fair prices the same constant of proportionality. This deviation simply accounts for the fact that the relative scarcity of the consumption good varies across states. The price of the claim that promises a delivery in the state with relatively low aggregate income would command a relatively high price. In that state, marginal utility of consumption would be high, so claims that deliver in low aggregate consumption states would be priced higher relative to their actuarially fair price.

9.1.4 Evidence on International Portfolio Diversification

We have made a case for international diversification of portfolios for two reasons. First, from the perspective of a mean–variance efficient portfolio, international diversification of portfolios has historically dominated a purely domestic portfolio. Second, from a risk-sharing perspective, individual countries should engage in asset trade to smooth their consumption across states. In this section, I will present empirical facts that speak to international portfolio diversification, and postpone discussion of empirical evidence on consumption smoothing until the next section.

We have seen that based on historical stock returns and return volatilities from 1970 to 1998, the mean–variance efficient portfolio from the Canadian investor’s perspective consists of about 75 percent international stocks and 25 percent Canadian stocks; see Figure 9.2. How does the average Canadian portfolio compare to this mean–variance efficient portfolio? Table 9.1 suggests that Canadian portfolio’s have historically been biased toward Canadian securities—while the efficient portfolio consists of 75 percent international stocks, average Canadian portfolios had less than 10 percent international stocks.

Evidence in Table 9.1 also suggests that Canadian portfolios are not exceptionally low in terms of their international content. For comparison, the mean–variance efficient portfolio for the U.S. is about 60 percent international, but the average portfolio has significantly less international share standing at about 12 percent in 2004 (Meirelles Aurélio, 2006). An average U.K. portfolio has significantly more international investments (about 32 percent in 1990), but for Germany, and Japan the portfolios are also highly biased toward domestic assets. Although I have not provided comparable evidence on the efficient portfolios for each of these countries, it is reasonable to conclude that there is significant “home bias” in financial portfolios (see also French and Poterba, 1991), although this bias has been declining over time (Meirelles Aurélio, 2006).

Since in principle international diversification in equity and bond markets yield higher returns at lower risk one possibility is to hold shares in multinational corporations traded in domestic markets. However, such a strategy does not yield the same level of diversification as holding an internationally diversified portfolio, because stocks are highly correlated with domestic factors, and thus do not provide sufficient diversification. In fact, stock market returns are more heavily influenced by domestic factors (such as

Table 9.1: International Diversification of Country Portfolios

<i>Country</i>	1970	1975	1980	1985	1990
<i>Canada</i>					
Portfolio investment	–	4.0	3.6	4.5	4.2
Stocks	–	7.1	6.0	6.5	6.6
Bonds	1.7	1.2	0.8	2.4	2.2
<i>Germany</i>					
Portfolio investment	4.9	2.4	2.7	5.8	10.2
<i>Japan</i>					
Portfolio investment	–	1.3	2.0	6.9	10.7
<i>UK</i>					
Portfolio investment	9.5	8.6	11.4	27.5	31.9
Stocks	–	–	16.9	24.8	23.5
Bonds	–	–	6.4	32.3	61.4
<i>US</i>					
Portfolio investment	–	2.3	2.2	2.2	2.7
Stocks	–	1.4	1.5	2.0	3.3
Bonds	2.6	3.0	2.8	2.4	2.4

Note: This table reports for each year the international financial investment positions of each country as shares of market capitalization values.

Source: Tesar and Werner (1995, Table 2).

monetary policy changes), than international factors (such as exchange rate movements). In other words, exchange rate movements and stock market returns are in general relatively weakly correlated, and therefore exchange rates do not provide a sufficient hedge against domestic market factors (i.e., $\gamma_{ik} \simeq 0$) See, e.g., Solnik (2000).

9.2 Intertemporal Consumption Smoothing

In the above section, I discussed how countries can insure against fluctuations in their marginal utility of consumption across states by risk pooling. The discussion was stylized because it had no recurrent uncertainty—hence no saving decisions, and there were no production decision. In reality, risk sharing has two dimensions: risk sharing across states of nature, and over time, whereby uncertainty is recurrent. And, risks can be mitigated using self-insurance mechanisms such as saving and investment in productive assets. In this section, I will introduce a model which will allow us to model saving decisions, without explicitly modeling investment decisions. However, I will briefly discuss the interactions between saving and investment decisions later on.

9.2.1 A Two-Period Model

Consider a two-period economy with a representative individual. The economy starts the first period with initial assets, which earn interest income (liabilities result in interest payments). For an open economy,

these assets correspond to net foreign assets. The individual also receives income in each period, and has to decide how much to consume and save in each period. Consequently, changes in net foreign assets, or the current account, reflect these saving decisions.

There is a single consumption good. The model with a single consumption good helps focus on the responses over time of consumption and saving decisions to changes in income and interest rates (*intertemporal* considerations).

Preferences.—In each period, the individual derives utility from a stream of consumption $(C(1), C(2))$, and maximizes the lifetime utility function

$$u(C(1)) + \beta u(C(2)). \quad (9.2.1)$$

The instantaneous utility function $u(C)$ is differentiable and strictly concave. This technical assumption allows us to characterize a unique consumption pair that maximizes lifetime utility subject to budget constraints (see below). Here, the first- and second-period instantaneous utility functions are identical. This is to simplify the exposition: for instance, when the utility function represents preferences over total consumption for a unit with changing demographics, such as a household.

In equation (9.2.1), $0 < \beta \leq 1$ is the subjective discount factor. An individual with a relatively high discount factor discounts future consumption at a low rate, and is relatively patient. Similarly, an individual with a low discount factor is relatively impatient, and would assign lower utility weight to future (second period) consumption.

Budget constraint.—Budget constraints reflect the amount of resources available for current and future consumption expenditures. For consumption purposes, a forward looking individual considers not only their current income and assets, but also their future income. Any unused income plus assets can be carried over into the future in the form of savings (say bonds).

Denote the beginning of period assets by B , which can be both positive or negative (in which case I refer to them as “debt”), the constant interest rate by r , and endowment income by Y . The timing of events is as follows. Each individual begins with initial assets, which earn a predetermined interest income over the period. At the end of the period, the individual receives income and determines the level of consumption expenditures C . The difference between the “cash at hand” $(1 + r)B + Y$ and consumption determines the beginning of next period assets, and so on. Thus, we have

$$\begin{aligned} B(2) &= (1 + r)B(1) + Y(1) - C(1), \\ 0 &= (1 + r)B(2) + Y(2) - C(2), \end{aligned} \quad (9.2.2)$$

where the last equality follows from the requirement that agents leave behind no outstanding debt (no bankruptcy condition) and no assets (no bequest motive). Initial asset holdings $B(1)$ is given. Figure 9.5

I will also define saving as the difference between disposable income and consumption

$$\text{Saving} = Y - C,$$

and use savings (plural) and “assets” or “debt” interchangeably.

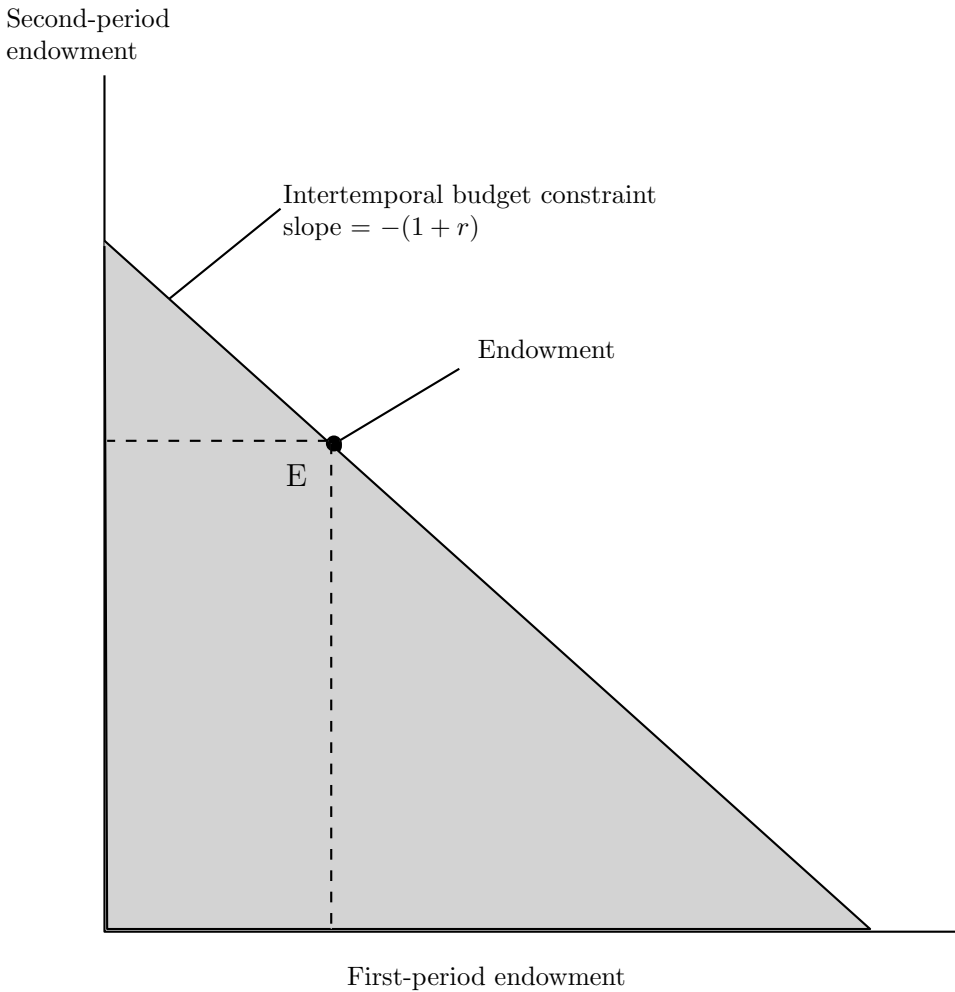


Figure 9.5: The intertemporal budget constraint: endowment economy

We can solve for $B(2)$ in the second budget constraint, and substitute it into the first one to obtain the *intertemporal* budget constraint:

$$C(1) + \frac{C(2)}{1+r} = (1+r)B(1) + Y(1) + \frac{Y(2)}{1+r}. \quad (9.2.3)$$

The right-hand-side of this equation is the present discounted value of current and future income plus beginning of period net foreign assets including interest payments. This determines the “lifetime” resources, which we assume exceed the present discounted value of subsistence consumption. The left-hand-side of the above equation determines the present discounted value of consumption. The two must be equal if the economy is to consume within its means. This expression also makes strong claims about financial markets: Lending and borrowing take place at the same world interest rate, and households can borrow fully against their future earnings—“perfect capital markets” assumption.

Equation (9.2.3) also suggests an alternative economic interpretation of the interest rate r . Normalize the price of first period consumption as to one. Then the relative price of second period consumption in terms of the first period consumption is

$$\frac{1}{1+r}.$$

Consequently, an increase in r leads to a decline in the relative price of second period consumption.

Optimality.—Let $(C(1), C(2)) > 0$ denote an optimal pair of first- and second-period consumption levels. Then, maximizing the objective function (9.2.1) with respect to the intertemporal budget constraint (9.2.3) gives the necessary condition for such an interior solution

$$u'(C(1)) = (1 + r)\beta u'(C(2)), \quad (9.2.4)$$

where $u'(C)$ denotes the marginal utility of consumption. (Given strict concavity of the instantaneous utility function, it turns out that the pair $(C(1), C(2))$, which satisfies the condition (9.2.4) is the unique optimal.) This condition states that marginal utility cost of saving one unit of good in the first period (the left-hand side) must be equal to the marginal utility benefit of that saving (the right-hand side). Marginal unit of the consumption good saved in the first period becomes $(1 + r)$ units of consumption good in the second period. Viewed from the first period, the marginal utility of second period consumption is $\beta u'(C(2))$. This argument also shows that consumption and saving decisions are inseparable, and involve tradeoffs across time periods—that is they are intertemporal decisions. Figure 9.6 demonstrates such an optimal consumption–saving decision.

The optimal consumption depends on lifetime resources. In fact, each period consumption can be thought of being financed by resources that resemble an annuity payment out of the present discounted value of lifetime wealth, or “permanent income.” While current income may fluctuate across period, permanent income, by construction, is constant.

Consumption-saving decisions in this set-up depend on two critical considerations. First, the desire to equate marginal utility of consumption (net of subsistence) over time determines the consumption smoothing motive for saving. This motive emerges in its starkest form when $\beta = 1/(1 + r)$. In this case, optimal consumption *levels* are identical over time, and saving emerges in response to fluctuating income— $Y(1) \neq Y(2)$, and literally reflects “saving for a rainy day.”

Second, saving in this setup depends on the magnitude of the interest rate relative to the discount rate. In particular, when $\beta(1 + r) \neq 1$, consumption levels are no longer identical across periods, and rather they are “tilted.” In this case, in addition to consumption smoothing motive, there is intertemporal substitution motive for saving. For instance, consider the case $\beta(1 + r) > 1$, whereby the effective discount factor is relatively “high.” In this case, the individual is willing to postpone consumption, so $C(1) < C(2)$. Technically this follows immediately from the fact that $u(C)$ is strictly concave, so the marginal utility of consumption is decreasing in C . In the opposite case of $\beta(1 + r) < 1$, the individual is relatively impatient, and consumption tends to decline over time.

Trade balance and the current account.—In this model, the trade balance TB is the difference between domestic income and domestic consumption:

$$TB(t) = Y(t) - C(t), \quad \text{for } t = 1, 2.$$

In the above expression and below, I use upper case Latin letters to denote aggregate variables, and lower case Latin letters to denote per person variables, including those of the representative individual.

The current account CA , on the other hand, can be expressed in two ways. First, since there are no unilateral transfers and worker’s remittances in this model, it is merely the sum of trade balance and interest

Second-period
endowment/consumption

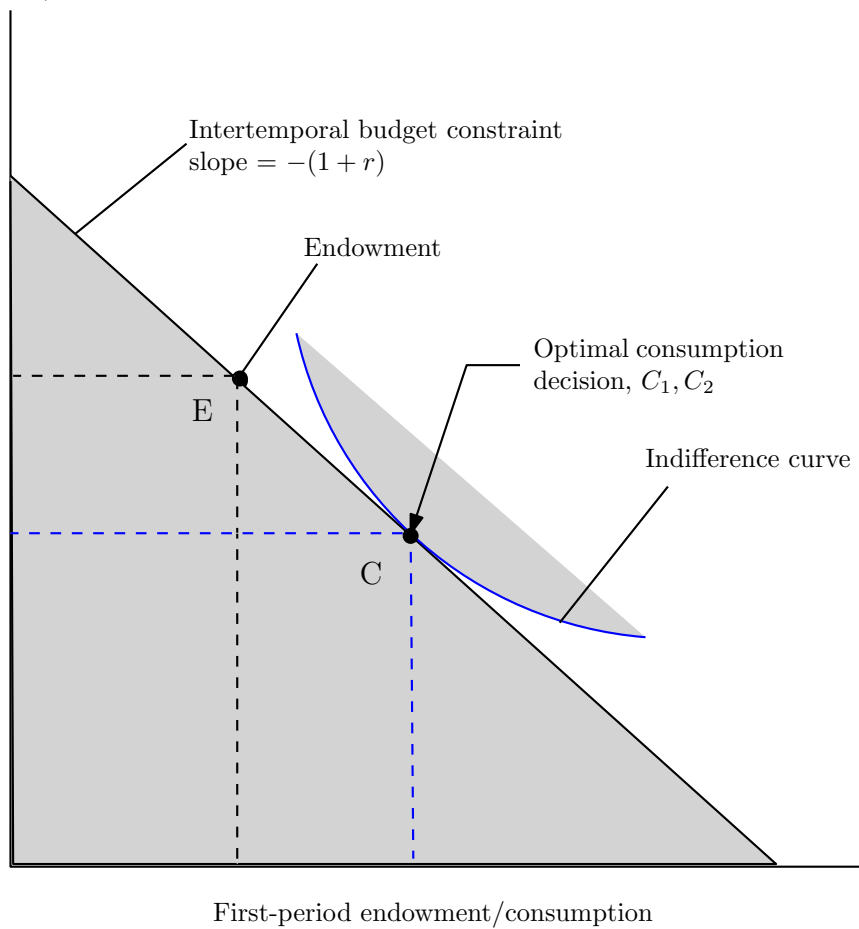


Figure 9.6: Consumption–saving decisions: endowment economy

earnings from abroad:

$$CA(t) = TB(t) + rB(t). \quad (9.2.5)$$

Second, the current account reflects the change in the net foreign assets position of a country. We can show the equivalence between the two definitions as follows.

$$\begin{aligned} CA(1) &= TB(1) + rB(1) && \text{(equation (9.2.5))} \\ &= Y(1) - C(1) + rB(1) && \text{(TB definition)} \\ &= Y(1) - C(1) + (1+r)B(1) - B(1) \\ &= B(2) - B(1), && (9.2.6) \end{aligned}$$

where the last line follows from the budget constraint (9.2.2). In the above expression $Y(1) + r \times B(1)$ is the gross national income. These alternative definitions have practical implications. Since, very few countries have reliable estimates for their net foreign assets, in practice, the balance on current account reported in the balance of payments accounts is almost always measured as the sum of balance on trade, net interest payments from abroad, workers' remittances, and unilateral transfers.

In the second and final period we have similar expressions for the current account

$$\begin{aligned} CA(2) &= Y(2) + rB(2) - C(2) \\ &= Y(3) - B(2). \end{aligned}$$

Note that outsiders would have no incentives to lend into the third period, so they would demand $B_3 \geq 0$. At the same time, residents would have no desire to leave unused resources into the final period. So, from their perspective $B_3 \leq 0$. The combination of these two incentives results in $B_3 = 0$.

The intriguing variable in the above analysis is the level of $B(1)$, the initial level of net foreign assets. In the small open economy model that I presented above, $B(1)$ is indeterminate—the model is entirely silent about the determinants of this variable. Yet, the level of $B(1)$ is critical for consumption and saving decisions.

To see this consider the case where $B(1) > 0$, so the country starts off with assets. What does this imply about its trade balance, and the current account? We can answer this by using the definitions of the current account (9.2.5) and ((equation (9.2.5))), and linking the trade balance to net foreign assets:

$$B(2) = TB(1) + (1 + r)B(1). \quad (9.2.7)$$

Next use the definitions of the current account adopted for period 2, to obtain a similar expression

$$B(3) = TB(2) + (1 + r)B(2). \quad (9.2.8)$$

Recall that the combination of no-bankruptcy condition and no desire to leave any assets into the final period implies $B(3) = 0$. So, combining (9.2.7) and (9.2.8), we have

$$(1 + r)B(1) = - \left(TB(1) + \frac{TB(2)}{1 + r} \right). \quad (9.2.9)$$

Thus, if a country starts off with foreign assets $B(1) > 0$, this could in practice help sustain a trade deficit in both periods. In fact, this generalizes to a multi-period accounting framework, whereby foreign assets can in principle finance trade deficits into perpetuity. In fact, the same conclusion holds for the current account, as long as the country meets the condition that its debt can be paid back with certainty in the distant future. On the other hand, if a country starts off with foreign debt $B(1) < 0$, its present discounted value of trade balances must be positive. Finally, consider the case where $B(1) = 0$. In this case, a trade surplus must be followed by a trade deficit and vice versa.

It is also instructive to mention the general context for trade balance and the current account, where there are investment and government expenditures. Let GDP denote the gross domestic product as in the national income and product accounts, GNP the gross national product (with $GNP = GDP + rB$), S domestic saving (private plus government), G government consumption expenditures and fixed investment, T taxes net of transfers, and I investment. Then, the current account CA is the difference between domestic saving and investment:

$$\begin{aligned} S &= \underbrace{(GNP - T - C)}_{\text{private saving}} + \underbrace{(T - G)}_{\text{public saving}}, \\ GNP &= C + G + I + CA, \\ CA &= S - I. \end{aligned}$$

Overall, in this two-period endowment economy model individuals and by extension economies respond to fluctuating aggregate income through savings. In other words, savings is a form of “self-insurance” against changes in income, although in this case these changes are perfectly foreseeable.

9.2.2 Evidence on Intertemporal Consumption Smoothing

In the context of risk sharing, international financial markets help countries smooth marginal utility of consumption across states. In the context of intertemporal consumption smoothing, international financial markets help countries smooth marginal utility of consumption over time. The intertemporal consumption smoothing hypothesis has several economically significant implications. For instance, when countries anticipate their income to increase in the *future*, the consumption smoothing hypothesis predicts that they would adjust their *current* consumption. Similarly, an anticipated decline in income would lead to an increase in current saving. In an international economy, these adjustments would be reflected in the movements in the current account.

An important implication of the intertemporal consumption smoothing hypothesis is that when a country increases investment spending due to a transitory increase in the rate of return to domestic capital, it can finance part of this investment spending through borrowing from abroad. As productive investment starts paying off in the future, the country can pay back the loans. This lending process thus allows the country to smooth consumption in several ways. First, it would not be required to increase current saving and thereby reduce current consumption to finance current investment spending. Second, by borrowing against higher future income. Jointly these two forces allow the country to smooth consumption. As such, it is possible to conceptualize one of the important functions of international financial markets as mediating this smooth consumption across time.

This very notion that countries can use international lending and borrowing as a way to smooth consumption has testable empirical implications. One implication of the hypothesis is that under capital mobility there should be a relatively low correlation between domestic saving and domestic investment rates. In fact, starting with Feldstein and Horioka (1980) economists have tested this hypothesis using econometric techniques. The following regression estimates are representative of the literature on this topic; see, e.g., Golub (1990), and Obstfeld and Rogoff (1996,162). The first set of estimates are essentially those reported by Feldstein and Horioka (1980). Their analysis involved relation saving–GDP ratios S/Y for 19 OECD countries to investment–GDP ratios I/Y . The next two sets of estimates show the changing saving–investment correlations across OECD countries over time.

$$\begin{aligned}
 1970\text{--}79 \ (N = 19): \quad I/Y &= 4.54 + 0.85 \ S/Y, \\
 &\quad (2.90) \quad (0.12) \\
 1980\text{--}86 \ (N = 19): \quad I/Y &= 4.65 + 0.74 \ S/Y, \\
 &\quad (4.10) \quad (0.18) \\
 1982\text{--}91 \ (N = 22): \quad I/Y &= 0.09 + 0.62 \ S/Y, \\
 &\quad (0.02) \quad (0.09)
 \end{aligned}$$

where N is the number of OECD countries. The results are striking in two respects. First, the correlations are “high”—ranging from 85 to 60 percent. Second, the correlations are progressively declining over time. Feldstein and Horioka’s original interpretation of these results was one of “limited capital mobility” across

OECD countries. They have argued that barriers to capital mobility were preventing these countries to take full advantage of international financial integration. Since Feldstein and Horioka, a large literature has examined whether these findings can be interpreted as evidence for limited capital mobility, and, if so, what are the ramifications of these results for consumption smoothing hypothesis. A more detailed discussion of these issues, however, must wait a more advanced course.

9.3 Recurrent Uncertainty

There is a substantial literature on international business cycles, which incorporates both recurrent uncertainty and production decisions, and investigates the implications of risk sharing in these more general contexts using two-country models. Generality comes at significant costs in several dimensions. First, often the only source of uncertainty in these models is “productivity.” Yet, it is unclear to what extent these productivity shocks are responsible for the observed fluctuations in income and consumption in reality. There is no compelling evidence to think that shocks to credit demand and supply, commodity price shocks, etc., are economically less important to think about the determinants of macroeconomic fluctuations. As such, conceptualizing the implications of risk sharing through productivity shocks alone is of limited practical use. Moreover, there is no consensus regarding whether macroeconomic fluctuations are driven by transitory or permanent shocks to the level or growth rate of productivity, or country-specific or common (global) shocks. We simply do not know whether one or a combination of these shocks is dominant in the actual data, and cannot be sure whether historical patterns will persist. Yet, our statements about the degree of actual risk sharing relative any benchmark depend squarely on these facts.

Nevertheless, if one is prepared to think productivity in a broad sense, capturing a variety of supply and demand shocks, it is possible to glean several insights based on these models. In this section, I discuss the basic implications of such a model with *complete* financial markets for cross-country correlations and the time-series properties of macroeconomic variables.

9.3.1 Theoretical Cross-Country Correlations

As a benchmark that delivers model-based correlations, I use a theoretical two-country model (see, e.g., Backus, Kehoe, and Kydland, 1992). Since building and solving such models are beyond the scope of these lectures, here I simply summarize their broader messages.

1. International risk sharing implies that consumption levels should highly correlated across countries. In other words, growth rates of consumption growth are (roughly) identical between any two pairs of countries (with and without asterisks)²

$$c(t) - c_{t-1} = \Delta c(t) = \Delta c^*(t).$$

2. International risk sharing implies that between any two country pairs, cross-correlations should be negative for output, investment i , and employment n , because resources flow into that country where they earn higher returns:

$$\rho_{y,y^*} < 0, \quad \rho_{i,i^*} < 0, \quad \rho_{n,n^*} < 0.$$

²All variables are logged and detrended.

3. When countries engage in international risk sharing through complete asset markets, transitory country-specific shocks should lead to the following configuration of the cross-country correlations ρ of output and consumption between any two pairs of countries (with and without asterisks)

$$\rho_{y,y^*} < \rho_{c,c^*}.$$

4. Net exports nx should be counter-cyclical:

$$\rho_{nx,y} < 0.$$

5. The response of net exports to a country-specific permanent productivity shock should exceed that of investment, because consumption also responds positively to such a productivity shock. So, the standard deviation of net exports should exceed that of investment:

$$\sigma(nx) > \sigma(i).$$

9.3.2 Evidence on International Consumption Smoothing

In this section, I will provide a discussion of the evidence that speaks to both drivers of international financial flows: risk sharing across states and risk sharing over time. Above, I have argued that viewed separately, there is weak evidence for sufficient portfolio diversification, and there is weak evidence for international capital mobility for sufficient intertemporal consumption smoothing. However, in the data, it is difficult to separate these two drivers, so it would be useful to see if the interactions between these two drivers can account for the empirical evidence. Such an encompassing exercise would require us to revisit cross-country correlations that allow for recurrent uncertainty.

Recall that under complete risk sharing, consumption correlations across countries should be higher than income correlations. This is perhaps the most significant implication of risk sharing under complete markets. The second important implication of complete risk sharing is that consumption *growth* rates should be highly correlated across countries. There is a large empirical literature that investigates whether these (and other) implications of the model hold in the data.

There is overwhelming evidence suggesting that consumption correlations across countries are actually lower than income correlations. In the data, consumption growth rates have low correlations across countries. Put together, these two pieces of evidence suggest that there is *imperfect* risk sharing across countries. In part, this finding is not surprising, because the assumption of complete international financial markets is not plausible. At the same time, there is an intriguing possibility that even under market incompleteness, countries may approximate perfect risk sharing through existing financial instruments (stocks, bonds, etc.). So, the positive implication of the empirical findings in light of the theoretical model is that existing market incompleteness leaves a substantial opportunity for welfare improvements in individual countries through international risk sharing.

In sum, there is two pieces of independent evidence suggesting that national portfolios have significant home bias, and that possibility due to this home bias significant international risk sharing opportunities remain unexploited. Whether and how economic policy might improve upon existing arrangements is beyond the scope of these lectures.