Technical Appendix for

Terms of Trade Risk with Partial Labor Mobility

Journal of International Economics
vol. 68, January 2006, pp. 92–114

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March 8, 2005

1 Solution of the Model

The following equations are also stated in the text:

$$q(L,Q) = A(L)^{\frac{1}{\nu}} Q^{\frac{\omega(1-\nu)}{\nu}} \tag{7}$$

$$\frac{dQ}{Q} = \mu \, dt + \sigma \, dz \tag{8}$$

$$w_f(Q) = \left[(1-\eta) + \eta Q^{\omega(\nu-1)}\right]^{\frac{1}{\nu-1}} \tag{9}$$

$$L_f(Q) = \left[1 + \left(\frac{1-\eta}{\eta}\right) Q^{\omega(1-\nu)}\right]^{-1} \tag{10}$$

$$\ln w_T(L, Q_T) - \ln w_N(L, Q_T) = \gamma S_T \tag{14}$$

$$\ln w_T(L, Q_N) - \ln w_N(L, Q_N) = \gamma S_N \tag{15}$$

1.1 Preliminaries

Traded versus non-traded relative price.— For P_T, we use the ideal price index:

$$\frac{P_T}{P_N} = \frac{P_{TF}^{\omega} P_{TH}^{1-\omega}}{P_N^{\omega} P_N^{1-\omega}} = q^{\omega-1} \left(Q \cdot q\right)^{-\omega} = q^{-1} Q^{-\omega}. \tag{1.1}$$

Relative sectoral wage.— To derive equation (7), start with the FOC:

$$C_T = C_N \left(\frac{\eta}{1-\eta}\right) \left(\frac{P_N}{P_T}\right)^{\nu}. \tag{1.2}$$

Market clearing states that $P_T C_T = P_{TH} Y_{TH}$ or:

$$P_T C_N \left(\frac{\eta}{1-\eta}\right) \left(\frac{P_N}{P_T}\right)^{\nu} = P_{TH} Y_{TH}. \tag{1.3}$$

Since $C_N = 1 - L$ and $Y_{TH} = L$, rearranging equation (1.3) gives:

$$\begin{aligned} \frac{1-L}{L} &= \left(\frac{1-\eta}{\eta}\right) \left(\frac{P_T}{P_N}\right)^{\nu-1} \left(\frac{P_{TH}}{P_N}\right) \\ &= \left(\frac{1-\eta}{\eta}\right) \left(q^{-1} Q^{-\omega}\right)^{\nu-1} q^{-1}. \end{aligned} \tag{1.4}$$

In equation (1.4), we used equation (1.1). Solving for q gives equation (7) in the text.

Employment shares without fixed relocation costs ("frictionless" case).— To obtain the expression for L_f, use equation (7) and set $q = 1$. Consequently:

$$\left(\frac{1-\eta}{\eta}\right)\left(\frac{L_f}{1-L_f}\right) = Q^{\omega(\nu-1)}. \tag{1.5}$$

Solving for L_f gives the desired equation (10).

Sectoral real wages.— For the non-traded real wage we have:

$$\begin{aligned}
\frac{W_N}{P} &= \frac{P_N}{\left[\eta P_T^{1-\nu} + (1-\eta)P_N^{1-\nu}\right]^{\frac{1}{1-\nu}}} \\
&= P_N\left[\eta P_{TF}^{\omega(1-\nu)} P_{TH}^{(1-\omega)(1-\nu)} + (1-\eta)P_N^{1-\nu}\right]^{\frac{1}{\nu-1}} \\
&= \left[\eta q^{\nu-1} Q^{\omega(\nu-1)} + (1-\eta)\right]^{\frac{1}{\nu-1}} \\
&= \left[\eta A(L)^{\frac{\nu-1}{\nu}} Q^{\gamma} + (1-\eta)\right]^{\frac{1}{\nu-1}}, \qquad (1.6)
\end{aligned}$$

where we used $q = A(L)^{\frac{1}{\nu}} Q^{\frac{\omega(1-\nu)}{\nu}}$, and $\gamma = \frac{\omega(\nu-1)}{\nu}$.

For the traded real wage, we have:

$$\begin{aligned}
\frac{W_T}{P} &= \frac{P_{TH}}{\left[\eta P_T^{1-\nu} + (1-\eta)P_N^{1-\nu}\right]^{\frac{1}{1-\nu}}} \\
&= P_{TH}\left[\eta P_{TF}^{\omega(1-\nu)} P_{TH}^{(1-\omega)(1-\nu)} + (1-\eta)P_N^{1-\nu}\right]^{\frac{1}{\nu-1}} \\
&= \left[\eta Q^{\omega(\nu-1)} + (1-\eta)q^{1-\nu}\right]^{\frac{1}{\nu-1}} \qquad (1.7) \\
&= \left[\eta + (1-\eta)A(L)^{\frac{1-\nu}{\nu}} Q^{-\gamma}\right]^{\frac{1}{\nu-1}} Q^{\omega}. \qquad (1.8)
\end{aligned}$$

Setting $q = 1$ in equation (1.7) gives frictionless real wage in equation (9) in the text.

1.2 Thresholds

To solve for the thresholds, we follow Dixit and Rob (1994) and express the expected benefits of relocating for an individual worker as the sum of two terms. First, a worker will calculate the expected present discounted value of the wage differential between the traded and non-traded sectors (U_Δ) under the assumption that there will be no further relocations in the future. Second, by relocating, a worker exercises (and therefore loses) the option of staying in the sector of origin and acquires in turn the option of staying in the destination sector. We will call this the net options value U_O, and it is also measured in utility terms. The sum of U_Δ and U_O is the relevant utility gain for a rational worker should he relocate. Both U_Δ and U_O are functions of Q and L.

First, consider the expected utility differential $U_\Delta(Q)$. For the current values of the terms of trade, Q_0, and sectoral allocation of labor, L, (since each worker takes this as a constant in competitive equilibrium), we have:

$$U_\Delta(Q_0) = \mathrm{E}\left[\int_0^\infty e^{-\rho t}\Delta(L, Q)\, dt\right]. \tag{1.9}$$

The dependence of U_Δ on L is implicit.

The expected discounted value of Δ is the solution $U_\Delta(Q_0)$ to the following differential equation (Harrison, 1985: 44–5):

$$\rho U_\Delta(Q_0) - \mu Q_0 U'_\Delta(Q_0) - \frac{\sigma^2}{2} Q_0^2 U''_\Delta(Q_0) = \Delta(L, Q_0).$$

Given Q_0, we can only solve this stochastic differential equation analytically for the function U_Δ:

$$U_\Delta(Q_0) = \left(\frac{1}{\rho}\right)\left[\gamma \ln Q_0 - \frac{1}{\nu}\ln A(L)\right] + \left(\frac{\gamma}{\rho^2}\right)\left(\mu - \frac{\sigma^2}{2}\right). \tag{1.10}$$

For a given current value of L, the expected discounted value of the wage gap between the traded and non-traded sectors increases in Q_0.

Second, consider the net options value of waiting to relocate. Relocation means that the worker loses the option of postponing the move. Because there is a dynamic uncertainty about the wage gap, waiting has an option value. Thus, whenever a decision to relocate takes place, this option is lost. Of course, after relocating the worker acquires another option: that of returning to the original sector. The net value of these two options comprises a non-dividend paying "asset", whose value depends purely on the revaluations (or "capital gains") arising from the uncertain changes in Q.

We denote this asset's utility value by $U_O(Q_0)$, which also depends on the current value of L. Over a time interval dt, the total expected return on the relocation opportunity (ρU_O) is equal to the expected capital gain on the net options:

$$\rho U_O(Q_0) = \frac{1}{dt}\mathrm{E}\left[dU_O(Q_0)\right].$$

Use Itô's Lemma to expand the right hand side of this equation to obtain:

$$\rho U_O(Q_0) - \mu Q_0 U'_O(Q_0) - \frac{\sigma^2}{2} Q^2 U''_{O_0}(Q_0) = 0.$$

The general solution to this equation is (Dixit and Pindyck, 1996: 140–44):

$$U_O(Q_0) = K_1(L) Q_0^{\beta_1} + K_2(L) Q_0^{\beta_2}, \tag{1.11}$$

where K_1 and K_2 are constants to be determined, and $\beta_1 > 0$ and $\beta_2 < 0$ are the roots of the quadratic equation (for any $\beta \in \mathbb{R}$):

$$\mathcal{Q}(\beta) \equiv \rho - \mu\beta - \frac{\sigma^2}{2}\beta(\beta - 1). \tag{1.12}$$

Combining equations (1.10) and (1.11), we determine the total utility gains associated with relocation: $U_\Delta + U_O$. For any given L, the critical values of Q that induce such a switch define *relocation thresholds*. These thresholds satisfy two optimality conditions known as "value-matching" and "smooth-pasting." The value-matching condition suggests that the total benefits of relocating must exceed the cost, c, of doing so. The smooth-pasting condition prevents large discrete changes in the value of the net options (i.e., large capital gains) at the time of its exercise that rational agents would already have arbitraged away [cf. Dixit and Pindyck (1996)].

There is one more issue that should be noted before we calculate the thresholds. The shape of these thresholds critically depends on the value of the elasticity of substitution between traded and non-traded goods ν. In the main text, we focus on the (empirically plausible) case where $\nu > 1$. For $\nu > 1$, the thresholds are increasing in the (L, Q) space. For $\nu < 1$, the thresholds are decreasing, but the analysis is entirely symmetric and we therefore omit it.

Relocation to the Traded Sector.— Consider first a worker located in the non-traded sector. Notice that for a non-traded worker, when $\nu > 1$, the likelihood of relocating increases with Q; see equation (7). Thus, the option is more likely to be exercised and command a positive value for larger values of Q. In other words, as Q increases, the value of the option increases and the option"moves into the money". Only the positive root β_1 delivers this result, so we set $K_2 = 0$. Let $Q_{T_1}(L)$ denote the value of terms of trade that optimally triggers the move from N to T for a given L (the subscript 1 is for the positive root of β). At the threshold, the value matching condition must be satisfied:

$$\left(\frac{1}{\rho}\right)\left[\gamma \ln Q_{T_1}(L) - \frac{1}{\nu}\ln A(L)\right] + \left(\frac{\gamma}{\rho^2}\right)\left(\mu - \frac{\sigma^2}{2}\right) + K_1(L) Q_{T_1}(L)^{\beta_1} = c.$$

The smooth-pasting condition, resulting from differentiating the value-matching condition with respect to Q, gives:

$$\left(\frac{1}{\rho}\right)\left(\frac{\gamma}{Q_{T_1}(L)}\right) + K_1(L)\beta_1 Q_{T_1}(L)^{\beta_1 - 1} = 0.$$

Using these two conditions to eliminate $K_1(L)$, we find the threshold value of terms of trade Q_{T_1} that will induce the worker to relocate:

$$Q_{T_1}(L) = \exp\left[S_{T_1} + \frac{1}{\omega(\nu - 1)}\ln A(L)\right], \tag{1.13}$$

where $S_{T_1} = \frac{\rho c}{\gamma} + \frac{1}{\beta_1} + \frac{1}{\rho}\left(\frac{\sigma^2}{2} - \mu\right)$.

Relocation to the Non-traded Sector.— Now consider a traded sector worker considering relocation to the non-traded sector. In this case, the value of the option increases as Q falls which requires us to use the negative root β_2 and set $K_1(L)$ to zero. Let $Q_{N_2}(L)$ denote the threshold value of terms of trade that optimally triggers the move from T to N for a given L (the subscript 2 is for the negative root of β). In this case, the value-matching condition is:

$$\left(\frac{1}{\rho}\right)\left[\gamma \ln Q_{N_2}(L) - \frac{1}{\nu}\ln A(L)\right] + \left(\frac{\gamma}{\rho^2}\right)\left(\mu - \frac{\sigma^2}{2}\right) + K_1(L) Q_{N_2}(L)^{\beta_2} = -c.$$

The smooth-pasting condition is identical (with Q_{N_2} instead of Q_{T_1}). Again, we use these conditions to eliminate $K_2(L)$ and find the threshold value of terms of trade $Q_{N_2}(L)$ that will induce the worker to relocate to the non-traded sector:

$$Q_{N_2}(L) = \exp\left[S_{N_2} + \frac{1}{\omega(\nu-1)}\ln A(L)\right], \tag{1.14}$$

where $S_{N_2} = -\frac{\rho c}{\gamma} + \frac{1}{\beta_2} + \frac{1}{\rho}\left(\frac{\sigma^2}{2} - \mu\right)$.

Finally, to obtain the (S, s) rules (14) and (15) given in the text, substitute the threshold equations (1.13) and (1.14) and the relative wage equation (7) in (1.8) and (1.6).

2 Solution of the Generalized Model

In this Appendix, we discuss the solution of the model when elasticity of substitution between home and foreign traded goods, ϱ, is different from one. The utility maximization problem can now be stated as:

$$\mathrm{E}\left[\int_0^\infty e^{-\rho t} \ln C_t \, dt\right] \tag{2.1}$$

subject to

$$C_t = \left[\eta^{\frac{1}{\nu}} C_{T,t}^{\frac{\nu-1}{\nu}} + (1-\eta)^{\frac{1}{\nu}} C_{N,t}^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}, \tag{2.2}$$

$$C_{T,t} = \left[\omega^{\frac{1}{\varrho}} C_{TF,t}^{\frac{\varrho-1}{\varrho}} + (1-\omega)^{\frac{1}{\varrho}} C_{TH,t}^{\frac{\varrho-1}{\varrho}}\right]^{\frac{\varrho}{\varrho-1}}. \tag{2.3}$$

Maximization of the consumer's objective function with respect to the budget constraint results in the following intra-temporal optimality conditions (dropping time subscripts):

$$\frac{C_{TF}}{C_{TH}} = \left(\frac{\omega}{1-\omega}\right)\left(\frac{P_{TH}}{P_{TF}}\right)^{\varrho}, \tag{2.4}$$

$$\frac{C_T}{C_N} = \left(\frac{\eta}{1-\eta}\right)\left(\frac{P_N}{P_T}\right)^{\nu}. \tag{2.5}$$

The consumption-based price indexes for composite traded good and composite good are (see Obstfeld and Rogoff [1996, p. 227]):

$$P_T = \left[(1-\omega) + \omega Q^{\varrho-1}\right]^{\frac{1}{1-\varrho}} P_{TH}, \tag{2.6}$$

$$P = \left[(1-\eta) + \eta\left(\frac{P_T}{P_N}\right)^{1-\nu}\right]^{\frac{1}{1-\nu}} P_N. \tag{2.7}$$

The non-traded real wage ($w_N = W_N/P$) and traded real wage ($w_T = W_T/P$) are given by:

$$w_N = \frac{P_N}{P} = \left[(1-\eta) + \eta\left(\frac{P_T}{P_N}\right)^{1-\nu}\right]^{\frac{1}{\nu-1}}, \tag{2.8}$$

$$\begin{aligned} w_T = \frac{P_{TH}}{P} &= \left[(1-\eta) + \eta\left(\frac{P_T}{P_N}\right)^{1-\nu}\right]^{\frac{1}{\nu-1}}\left(\frac{P_{TH}}{P_N}\right), \\ &= \left[(1-\eta) + \eta\left(\frac{P_T}{P_N}\right)^{1-\nu}\right]^{\frac{1}{\nu-1}}\left(\frac{1}{q}\right). \end{aligned} \tag{2.9}$$

Clearly, in an economy without fixed relocation costs, $q = 1$, and the two are identical. In an economy with relocation costs, however, the relative wage, q is not necessarily one and is (as in the unitary elasticity case):

$$q(L,Q) = A(L)\left(\frac{P_T}{P_N}\right)^{\nu-1}. \tag{2.10}$$

The relative traded price is:

$$\begin{aligned} \frac{P_T}{P_N} &= \left[(1-\omega) + \omega Q^{\varrho-1}\right]^{\frac{1}{1-\varrho}}\left(\frac{P_{TH}}{P_N}\right), \\ &= \left[(1-\omega) + \omega Q^{\varrho-1}\right]^{\frac{1}{1-\varrho}}\left(\frac{1}{q}\right). \end{aligned} \tag{2.11}$$

Now define:

$$\xi = \frac{(\nu-1)}{\nu(\varrho-1)}, \qquad \text{and} \qquad Q^* = \left[(1-\omega) + \omega Q^{\varrho-1}\right].$$

Thus, using equation (2.11) in equations (2.8) and (2.9), the frictionless real wage ($w_f = W_f/P$) becomes:

$$w_f(Q^*) = \left[(1-\eta) + \eta\,(Q^*)^{\frac{\nu-1}{\varrho-1}}\right]^{\frac{1}{\nu-1}}. \tag{2.12}$$

Similarly, using equation (2.11) in equation (2.10), the relative wage is:

$$q(L, Q^*) = A(L)^{1/\nu} Q^{*-\xi}. \tag{2.13}$$

Solving for the frictionless labor allocation for $q = 1$ gives:

$$L_f(Q^*) = \left[1 + \left(\frac{1-\eta}{\eta}\right) Q^{*\frac{\nu-1}{1-\varrho}}\right]^{-1}. \tag{2.14}$$

The welfare measure of the real wage gap is given by:

$$\begin{aligned}\Delta(L, Q^*) &= \ln w_T - \ln w_N \\ &= -\ln q \\ &= \xi \ln Q^* - \frac{1}{\nu}\ln A(L). \end{aligned} \tag{2.15}$$

where in the third line we used equation (2.13).

Note that from Result 1 in Appendix 3 the drift and variance of the diffusion process for Q^* is given by:

$$\mu^*(Q) = \Gamma^*(Q)\left[\mu + \frac{\sigma^2}{2}(\varrho-2)\right], \tag{2.16}$$

$$\sigma^*(Q) = \Gamma^*(Q)\sigma, \tag{2.17}$$

where

$$\Gamma^*(Q) = \left(\frac{\omega Q^{\varrho-1}}{(1-\omega)+\omega Q^{\varrho-1}}\right)(\varrho-1).$$

As discussed in the text, we assume that the worker evaluates the expected PDV by treating these moments as fixed given the current period realization of Q. Of course, once a new value of the terms of trade is observed, the worker updates these moments.

Solving for the thresholds is identical to the case from Appendix 1 (simply substitute ξ for γ). For moves from the non-traded to the traded goods sector, the threshold Q^* value is:

$$Q^*_{T_1}(L,Q) = \exp\left[S^*_{T_1}(Q) + \left(\frac{\varrho-1}{\nu-1}\right)\ln A(L)\right], \tag{2.18}$$

where the subscript 1 refers to the fact that we are using the positive β^* root from the quadratic $\rho - \mu^*(Q)\beta^*(Q) - \frac{\sigma^*(Q)^2}{2}\beta^*(Q)\left[\beta^*(Q)-1\right]$, and $S^*_{T_1}(Q) = \frac{\rho c}{\xi} + \frac{1}{\beta^*_1(Q)} + \frac{1}{\rho}\left[\frac{\sigma^*(Q)^2}{2} - \mu^*(Q)\right]$. Likewise, for moves from the traded to the non-traded goods sector, the threshold Q^* value is:

$$Q^*_{N_2}(L,Q) = \exp\left[S^*_{N_2}(Q) + \left(\frac{\varrho-1}{\nu-1}\right)\ln A(L)\right], \tag{2.19}$$

where the subscript 2 refers to the fact that we are using the negative β^* root from the fundamental quadratic, and $S^*_{N_2}(Q) = -\frac{\rho c}{\xi} + \frac{1}{\beta^*_2(Q)} + \frac{1}{\rho}\left[\frac{\sigma^*(Q)^2}{2} - \mu^*(Q)\right]$.

To find the percentage (real) wage gaps that correspond to the triggering threshold values of Q^*, we proceed as follows. Notice that after substituting equation (2.13) into (2.8) and (2.9), the real wages in the non-traded and traded sectors can be rewritten as:

$$w_N(L,Q^*) = \left[(1-\eta) + \eta A(L)^{\frac{\nu-1}{\nu}} Q^{*\frac{1-\nu}{1-\varrho}+\xi(1-\nu)}\right]^{\frac{1}{\nu-1}}, \tag{2.20}$$

$$w_T(L,Q^*) = \left[(1-\eta) + \eta A(L)^{\frac{\nu-1}{\nu}} Q^{*\frac{1-\nu}{1-\varrho}+\xi(1-\nu)}\right]^{\frac{1}{\nu-1}} A(L)^{\frac{-1}{\nu}} Q^{*\xi}. \tag{2.21}$$

The percentage wage gap is

$$\ln w_T(L,Q^*_{T_1}) - \ln w_N(L,Q^*_{T_1}) = -\frac{1}{\nu}\ln A(L) + \xi \ln Q^*_{T_1}(L,Q).$$

Since

$$\xi \ln(Q^*_{T_1}(L,Q)) = \xi S^*_{T_1}(Q) + \frac{1}{\nu}\ln A(L),$$

the percentage wage gap triggering moves into the traded sector is

$$\ln w_T(L,Q^*_{T_1}) - \ln w_N(L,Q^*_{T_1}) = \xi S^*_{T_1}(Q). \tag{2.22}$$

For moves from the traded sector into non-traded, we have:

$$\ln w_N(L,Q^*_{N_2}) - \ln w_T(L,Q^*_{N_2}) = \frac{1}{\nu}\ln A(L) - \xi \ln Q^*_{N_2}(L,Q).$$

Since

$$\xi \ln(Q^*_{N_2}(L, Q)) = \xi S^*_{N_2}(Q) + \frac{1}{\nu} \ln A(L),$$

the percentage wage gap triggering moves into the non-traded sector is

$$\ln w_T(L, Q^*_{N_2}) - \ln w_N(L, Q^*_{N_2}) = \xi S^*_{N_2}(Q). \tag{2.23}$$

3 Conditional Covariances

In what follows, we make repeated use of the following two results.

Result 1: Let dQ be given by equation (8), and $F(Q) = (a + bQ^g)^h$. Then, using Itô's Lemma, we have:

$$\frac{dF}{F} = \mu_F(Q)\, dt + \sigma_F(Q)\, dz,$$

where:

$$\begin{aligned}
\mu_F(Q) &= \Gamma(Q) \left\{ \mu + \frac{\sigma^2}{2} \left[(g-1) + \Gamma(Q) \left(1 - \frac{1}{h} \right) \right] \right\}, \\
\sigma_F(Q) &= \Gamma(Q)\sigma, \\
\Gamma(Q) &= \left(\frac{bQ^g}{a + bQ^g} \right) \cdot g \cdot h.
\end{aligned}$$

The instantaneous variance rate of the changes in F is $[\sigma_F F]^2$, or alternatively the instantaneous variance of the growth rate of F is σ_F^2.

Result 2: Let dQ be given by equation (8), and $F(Q) = G(Q) \cdot H(Q)$, where G and H are both Itô processes with differentials:

$$\begin{aligned}
dG &= \mu_G G dt + \sigma_G G dz, \\
dH &= \mu_H H dt + \sigma_H H dz,
\end{aligned}$$

where both are built from a common standard Brownian motion z. Thus, using the "chain rule" (Harrison 1985, p.72), we have:

$$dF = GdH + HdG + (dG)(dH),$$

where $(dG)(dH) = \sigma_G \sigma_H F dt$. After substituting the expressions for dG and dH, we obtain:

$$\frac{dF}{F} = [\mu_G + \mu_H + \sigma_G \sigma_H]\, dt + [\sigma_G + \sigma_H]\, dz.$$

We now turn to the calculation of instantaneous variances and covariances.

1. *Frictionless Real Wage* (σ_f).— Follows from the immediate application of result 1 to equation (9).

2. *Non-traded Real Wage* (σ_N).— Apply the result 1 to the non-traded real wage equation:

$$w_N = \left[(1-\eta) + \eta A(L)^{\frac{\nu-1}{\nu}} Q^{\gamma}\right]^{\frac{1}{\nu-1}}, \tag{3.1}$$

which gives:

$$\begin{aligned} \mu_N(Q) &= \Gamma_N(Q)\left\{\mu + \frac{\sigma^2}{2}\left[(g_N - 1) + \Gamma_N(Q)\left(1 - \frac{1}{h_N}\right)\right]\right\}, & (3.2)\\ \sigma_N(Q) &= \Gamma_N(Q)\sigma, & (3.3)\end{aligned}$$

where:

$$\begin{aligned} a_N &= 1-\eta, \quad b_N = \eta A(\overline{L})^{\frac{\nu-1}{\nu}} \\ g_N &= \gamma, \qquad h_N = \frac{1}{\nu-1}, \\ \Gamma_N(Q) &= \left(\frac{b_N Q^{g_N}}{a_N + b_N Q^{g_N}}\right) g_N h_N. \end{aligned}$$

3. *Traded Real Wage* (σ_T).— Use the traded real wage equation:

$$w_T(L,Q) = \left[\eta + (1-\eta)A(L)^{\frac{1-\nu}{\nu}} Q^{-\gamma}\right]^{\frac{1}{\nu-1}} Q^{\omega}, \tag{3.4}$$

and let $w_T = G_T H_T$ where:

$$\begin{aligned} G_T &= \left[\eta + (1-\eta)A(L)^{\frac{1-\nu}{\nu}} Q^{-\gamma}\right]^{\frac{1}{\nu-1}} \\ H_T &= Q^{\omega}. \end{aligned}$$

Using result 2, we get:

$$\frac{dw_T}{w_T} = \mu_T dt + \sigma_T dz,$$

where $\mu_T = \mu_{G_T} + \mu_{H_T} + \sigma_{G_T}\sigma_{H_T}$, and $\sigma_T = \sigma_{G_T} + \sigma_{H_T}$.

To find μ_{G_T} and σ_{G_T} use result 1:

$$\begin{aligned} \mu_{G_T}(Q) &= \Gamma_{G_T}(Q)\left\{\mu + \frac{\sigma^2}{2}\left[(g_{G_T} - 1) + \Gamma_{G_T}(Q)\left(1 - \frac{1}{h_{G_T}}\right)\right]\right\}, & (3.5)\\ \sigma_{G_T}(Q) &= \Gamma_{G_T}(Q)\sigma, & (3.6)\end{aligned}$$

where:

$$\begin{aligned}
a_{G_T} &= \eta, \qquad b_{G_T} = (1-\eta)A(\overline{L})^{\frac{1-\nu}{\nu}}, \\
g_{G_T} &= -\gamma, \quad h_{G_T} = \frac{1}{\nu - 1}, \\
\Gamma_{G_T}(Q) &= \left(\frac{b_{G_T} Q^{g_{G_T}}}{a_{G_T} + b_{G_T} Q^{g_{G_T}}} \right) g_{G_T} h_{G_T}.
\end{aligned}$$

To find μ_{H_T} and σ_{H_T} use Itô's Lemma:

$$\mu_{H_T} = \omega\mu + \frac{1}{2}\omega(\omega - 1)\sigma^2, \tag{3.7}$$

$$\sigma_{H_T} = \omega\sigma. \tag{3.8}$$

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